Mean Photon Numbers of Coherent and Squeezed Light States
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1 Introduction

Coherent and squeezed states are the two canonical families of nonclassical optical field states generated from the bosonic mode algebra and Gaussian unitaries. Coherent states are the closest quantum analogue of a classical monochromatic field: they are displaced vacuum states with Poissonian photon-number statistics and a well-defined complex field amplitude, making them the natural reference for laser light and for semiclassical detection theory. Squeezed states, by contrast, redistribute quantum fluctuations between conjugate field quadratures while preserving the Heisenberg uncertainty bound; they enable sub-shot-noise measurements, improved phase sensitivity, and noise reduction in interferometry [1]. In a single mode, squeezing can be viewed as a Bogoliubov (linear canonical) transformation that mixes annihilation and creation operators, thereby producing a state with zero mean field but modified quadrature variances and a nonzero mean photon number [2, 3].

2 Mean Photon Number

The photon-number (number) operator for a single mode is defined as

$$n \equiv a^\dagger a \tag{1}$$

Using the canonical commutator relation (CCR), $[a, a^\dagger] = 1$, we find the useful identities

$$[n, a] = -a, \qquad [n, a^\dagger] = a^\dagger \tag{2}$$

which express that a lowers and $a^\dagger$ raises the photon number by one unit. The Fock (number) states are constructed from the vacuum $|0\rangle$ (defined by $a|0\rangle = 0$) as

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}\,|0\rangle, \qquad n = 0, 1, 2, \ldots \tag{3}$$

and satisfy the eigenvalue equation

$$n\,|n\rangle = n\,|n\rangle \tag{4}$$

Finally, for an arbitrary state $|\psi\rangle$ the *mean photon number* is defined as the expectation value of the number operator,

$$\langle n\rangle_\psi \equiv \langle\psi|\,a^\dagger a\,|\psi\rangle = \sum_{n=0}^{\infty} n\,|\langle n|\psi\rangle|^2 \tag{5}$$

where the last expression follows by inserting the resolution of the identity in the Fock basis.

2.1 Coherent Light

For $|\alpha\rangle = D(\alpha)|0\rangle$ with Bogoliubov transform $D^\dagger a D = a + \alpha$ (for details see "From Generators to Fock-Space Superpositions: Coherent and Squeezed Light" primer), we obtain

$$\begin{aligned}\langle n\rangle_\alpha &= \langle D(\alpha)0|\, a^\dagger a\, |D(\alpha)0\rangle \\ &= \langle 0|\, D^\dagger(\alpha) a^\dagger a D(\alpha)\, |0\rangle \\ &= \langle 0|\, D^\dagger(\alpha) a^\dagger D(\alpha) D^\dagger a D(\alpha)\, |0\rangle \\ &= \langle 0|(a^\dagger + \alpha^*)(a+\alpha)|0\rangle \\ &= \boxed{|\alpha|^2.}\end{aligned} \tag{6}$$

where we have used "ket to bra" conversion, and

$$(D^\dagger a D)^\dagger = D^\dagger a^\dagger D = (a+\alpha)^\dagger = a^\dagger + \alpha^* \tag{7}$$

2.2 Squeezed Light

Let $|0;\zeta\rangle = S(\zeta)|0\rangle$ be the squeezed vacuum and we apply the Bogoliubov transform (see primer "From Generators to Fock-Space Superpositions: Coherent and Squeezed Light") [4]:

$$\begin{aligned} S^\dagger(\zeta)\, a\, S(\zeta) &= \mu\, a + \nu\, a^\dagger \\ S^\dagger(\zeta)\, a^\dagger\, S(\zeta) &= \mu^* a^\dagger + \nu^* a \\ |\mu|^2 - |\nu|^2 &= 1 \\ \mu &= \cosh r, \\ \nu &= e^{i\theta} \sinh r \end{aligned} \tag{8}$$

This constraint ensures that $[a, a^\dagger] = 1$ is preserved under the transform. Then the mean photon number is

$$\begin{aligned}\langle n\rangle_\zeta &\equiv \langle 0;\zeta|a^\dagger a|0;\zeta\rangle \\ &= \langle 0|\, S^\dagger(\zeta)\, a^\dagger a\, S(\zeta)\, |0\rangle \\ &= \langle 0|\big(S^\dagger a^\dagger S\big)\big(S^\dagger a S\big)|0\rangle \\ &= \langle 0|(\mu^* a^\dagger + \nu^* a)(\mu a + \nu a^\dagger)|0\rangle.\end{aligned} \tag{9}$$

Expanding and using $a|0\rangle = 0$ gives

$$\langle n\rangle_\zeta = |\nu|^2\, \langle 0|a a^\dagger|0\rangle = |\nu|^2, \tag{10}$$

since $\langle 0|a a^\dagger|0\rangle = 1$ while all other vacuum expectations vanish. With $\mu = \cosh r$ and $\nu = -e^{i\theta} \sinh r$ (or the equivalent phase convention),

$$\boxed{\langle n\rangle_\zeta = \sinh^2 r.} \tag{11}$$

3 Conclusion.

Once the Bogoliubov (linear canonical) transformation is known, the mean photon number follows almost immediately by conjugating $a^\dagger a$ through the unitary and evaluating on the vacuum. For the squeezed vacuum, all terms vanish except the $aa^\dagger$ contribution, leaving $\langle n\rangle_\zeta = |\nu|^2 = \sinh^2 r$, which makes the physical content transparent: squeezing populates the mode by mixing creation and annihilation operators, with the squeezing strength r directly setting the photon-number scale. The coherent-state case provides a complementary picture: displacement shifts the field by a c-number and yields $\langle n\rangle_\alpha = |\alpha|^2$. Together, these results illustrate a general strategy used throughout quantum optics: rather than expanding states in the Fock basis, many expectation values can be computed efficiently by transforming the operators and exploiting simple vacuum matrix elements.

References

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[3] M Scully and M S Zubairy. *Quantum optics*. Cambridge University Press, 1997.

[4] D F Walls and G J Milburn. *Quantum Optics*. Springer Nature Switzerland, 2 edition, 2008.