Chapter 10: Quantum Subsystems and Properties of Entangled States

Overview

In this section, we will investigate quantum subsystems, entangled states, their characterization, and provide the groundwork for error correction codes

Learning Objectives

By the end of this section, you should be able to:

1. Describe the difference between superposition, pure and mixed states.
2. Distinguish density matrices for pure and mixed states.
3. Describe the concept of superoperators in the context of entanglement.

Chapter 10:

Quantum Subsystems and Entangled States.

Power of Quantum Computing:

- n-qubits $\to$ 2^n states

  $n = 50 \to \sim 10^{15}$ states

  Google 2019: 53-qubits:

  (10^4 years $\to$ 3m22s)

- <u>measurement</u>: only can measure n states! Limited probing of state space.

- Entanglement necessary for exponential speed-up. (Josza & Linden, 2003)

- Entanglement generally poorly understood, especially for large n (many qubits).

$\Rightarrow$ Questions: (Chapter 10)

- How to characterize entanglement?

- Interaction of subsystems.

$$\underbrace{\underbrace{|\psi\rangle = |\text{Quantum}\rangle}_{\text{Computing}} \otimes |\text{Environment}\rangle}_{\text{decoherence \& error.}}$$

Review entangled states:

n-qubit state

$\neq$ tensor product of

n 1-qubit states

$\Rightarrow$ entangled.

Ex: Bell-state:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\stackrel{?}{=} \frac{1}{\sqrt{2}}\left(a_1|0\rangle + b_1|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(a_2|0\rangle + b_2|1\rangle\right)$$

$$= \frac{1}{2}\left(a_1a_2|00\rangle + a_1b_2|01\rangle + b_1a_2|10\rangle + b_1b_2|11\rangle\right)$$

$\Rightarrow a_1b_2 = 0 \wedge b_1a_2 = 0$

$\Rightarrow$ impossible!

Similar for $|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$

Consequences:

EPR - paradox: (Einstein et al. 1935)

Alice: 1^{st} qubit
Bob: 2^{nd} qubit

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

separate systems:

Alice measures "0" $\rightarrow$ Bob: "0"

"1" $\rightarrow$ "1"

suggests: instanteneous knowledge:

$v \rightarrow \infty \gg c$

Einstein, Podolsky, Rosen: (1935)

cannot be $\rightarrow$ need other solution.

EPR: hidden variables.

Quantum Mechanics is incomplete!

~~Bohr (1935) incorrect conclusion~~.

Bohr (1935)

incorrect conclusion.

Bob can measure any direction but $\{\hat{A}, \hat{B}\} \neq 0$

$\Rightarrow$ Bob's measurement will be "uncertain" probabilistic:

Alice: S_z : "0" "1"

Bob: S_x : 50%/50% 50/50

only possible if Alice sends direction through classical channel $\leq c$.

$\Rightarrow$ quantum mechanics is complete.

Bell 1960ies + later experiments confirm Bohr's argument and conclusion.

⇒ <u>no-cloning theorem.</u>

unknown quantum state cannot be cloned/copied using any unitary operator.

⇒ <u>teleportation</u>

Alice — Bob

$|\phi\rangle|0\rangle \longrightarrow |x\rangle|\phi\rangle$

initial state (Alice qubit) destroyed.

only one person @ a time can re-construct quantum state!

The wave function of a state contains all possible knowledge about the system: $\boxed{|\psi\rangle \text{ is complete.}}$

Entanglement depends on decomposition.

unentangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \otimes |0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|000\rangle + |110\rangle\right)$$

$$\stackrel{?}{=} \left(a_1|0\rangle + b_1|1\rangle\right) \otimes \left(a_2|0\rangle + b_2|1\rangle\right) \otimes \left(a_3|0\rangle\right)$$

$$\cancel{= a_1 a_2 a_3 |000\rangle + a_1 b_2 a_3 |010\rangle + b_1 a_2 a_3 +}$$

entangled

⇒ state is entangled wrt to 1-qubit computational basis.

In general:

$$2^n \gg n$$

⇒ most states are entangled

⇒ chance for quantum computing to show exponential speed-up (Josza & Linden, 2003)

Characterizing entangled states:

- superposition: $|\psi\rangle = a_1|0\rangle + b_1|1\rangle$ ---

  ~~chapter (1–4)~~; forms basis of quantum mechanics.

- pure states: (chapters 1–9)

  well defined states:

  $V = V_1 \otimes V_2 \otimes \cdots \otimes V_n$

  all single elements of tensor product space.

- does not mean unique wrt. measurement.

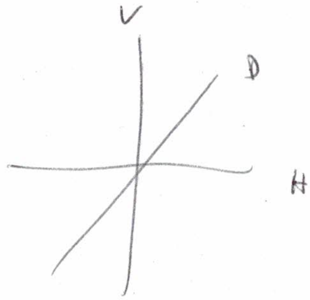
  we know all coefficients of all states that occur.

  $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$

  is a pure state.

Example:

Photon:



Before measurement: well-defined! = pure

$|\text{Photon}\rangle = |D\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$

after measurement:

$$\left.\begin{array}{l} 50\%\ |H\rangle \\ 50\%\ |V\rangle \end{array}\right\} \text{ but certain uncertain.}$$

Mixed states:

simply a set of (independent) pure states:

usually this implies some uncertainty about the system.

$$|\psi\rangle = \underbrace{|\text{quantum}\rangle \underbrace{\otimes\ |\text{environment}\rangle}_{\text{uncertain.}}}_{\text{decoherence.}}$$

$\Rightarrow$ need error correction.
Chapter 11.

Measurement: Neilsen & Chuang (chapter 2)

Quantum mechanics, postulate:

collection of measurement operators:

$$\{\hat{M}_m\}$$

$$\uparrow$$

measurement index

Before measurement: $|\psi\rangle$

$\Rightarrow$ probability to measure m:

$$p(m) = \langle\psi|\,\hat{M}_m^{+}\,\hat{M}_m\,|\psi\rangle \in \mathbb{R}$$

and state after measurement is

$$|\psi'\rangle = \frac{M_m|\psi\rangle}{\sqrt{\langle\psi|\,\hat{M}_m^{+}\,\hat{M}_m\,|\psi\rangle}}$$

completeness:

$$\sum \hat{M}_m^{+}\,\hat{M}_m = \mathbb{1}$$

$$\sum p(m) = 1$$

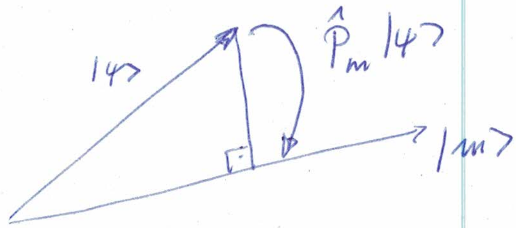
Projectors:

$$\hat{M} = \sum_m m \hat{P}_m$$ = sum over all possible measurement outcomes.

$$p(m) = \langle \psi | \hat{P}_m | \psi \rangle$$

$$|\psi\rangle = \frac{\hat{P}_m |\psi\rangle}{\sqrt{p(m)}}$$

Moreover: ~~classical~~ $\boxed{\hat{P}_m^2 = \hat{P}_m}$



$$= \hat{P}_m (\hat{P}_m |\psi\rangle) = \hat{P}_m^2 |\psi\rangle$$

$\Rightarrow$ only one operator appears in $p(m)$ not a product of two operators as before.

A useful quantity:

$\{m\}$ are possible outcomes

$$\Rightarrow \quad \mathbb{1} = \sum_m |m\rangle\langle m|$$

reflects that we have a complete knowledge of all possible out_comes

and in general: as in linear algebra:

matrix version of operators $\hat{A}$:

~~a_{ij}~~

$$a_{ij} = \langle i|\hat{A}|j\rangle$$