Quantum Information Science (QIS)

Dr. Boris Kiefer, Lecture 1

Quantum Computing
Quantum Communication
Quantum Sensing

QIS exploits quantum principles to transform how information is acquired, encoded, manipulated, and applied. QIS encompasses quantum computing, quantum communication, and quantum sensing.

1. QIS employs quantum mechanics, a well-tested theory that uses the mathematics of probability, vectors, algebra, and linear transformations to describe the physical world.

2. QIS combines information theory and computer science.

3. QIS demonstrated impact on high-impact technologies, such as GPS which depends on the extreme precision of atomic clocks.

Announcements:

Review:	02/21/2022
Midterm 1:	02/23/2022
Title and Abstract for projects:	03/18/2022

Lecture Content: 02/14/2022 & 02/16/2022

- A short history of the electron.
- Spin, a quantum discovery.
- Two types of particles: fermions and bosons.
- The Periodic Table of Elements.

Quantum Information Science (QIS):

Inputs:
- Quantum states.
- Measurements.
- Qubits.
- Entanglement.
- Decoherence.

Outputs:
- Quantum Communication.
- Quantum Sensing.
- Quantum Computing.

Lecture Content: 02/14/2022 & 02/16/2022

Quantum Computing:

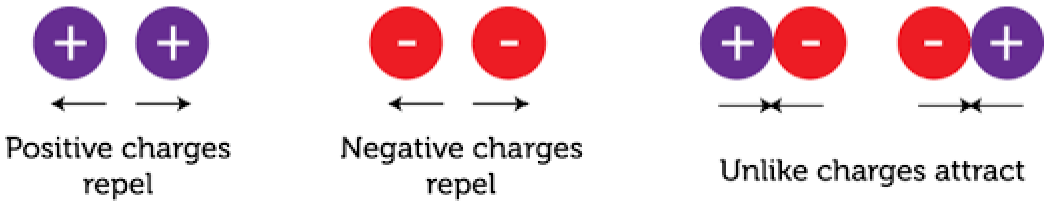
- Some physics of superconducting quits:
  - Josephson junctions.
  - Logic gates: SWAP and CNOT.
  - Entanglement.
  - Current status, challenges and opportunities.
- Other qubit realizations.

Quantum Computing: Theory and Practice:

- IBM-Q: Quisket.
- Quantum states on a quantum processor: GHZ and Dicke states.
- Teleportation and telecloning.

Memory Lane

A Very Short History of the Electron



++//-- charge combination **move** apart.
+-//-+ charges **move** closer.

Expectation: interacting charges move.

Mechanical motion:

Sir Isaac Newton (1642-1726)

F = m a

m a = F

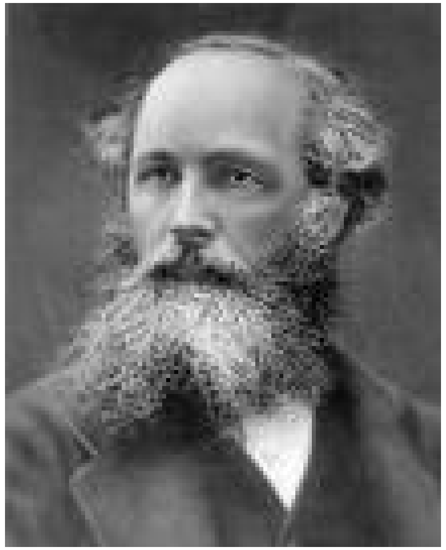
motion acceleration ↑ ... ↑ force

➔ Need force.

Electricity, 18th + 19th Century

Charles Coulomb
(1736 – 1806)

$$E_{Coulomb} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 \cdot q_2}{R}$$



James Clerk Maxwell
(1831 – 1879)

Electrodynamics

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

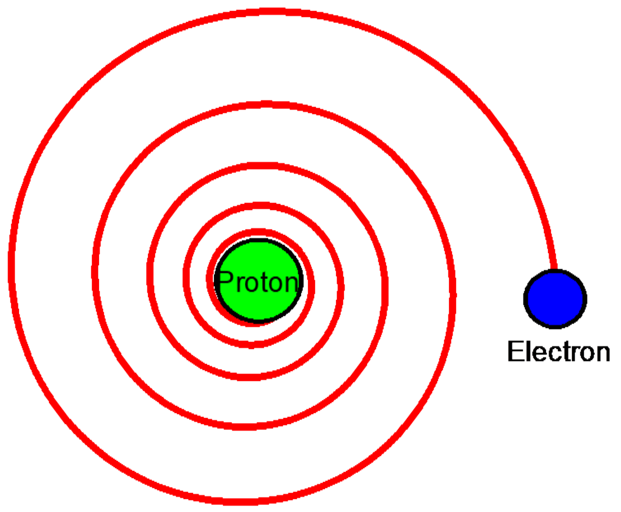
http://www.microwaves101.com/encyclopedia/images/maxwellsequations/maxwellsequations.jpg
http://upload.wikimedia.org/wikipedia/et/thumb/b/ba/Charles_Coulomb.jpg/250px-Charles_Coulomb.jpg
http://outreach.jach.hawaii.edu/pressroom/2006_jcmbday/Maxwell.jpg

"The Paradox of Everything"

- Electron (Thomson, 1897):
  negatively charged particle.

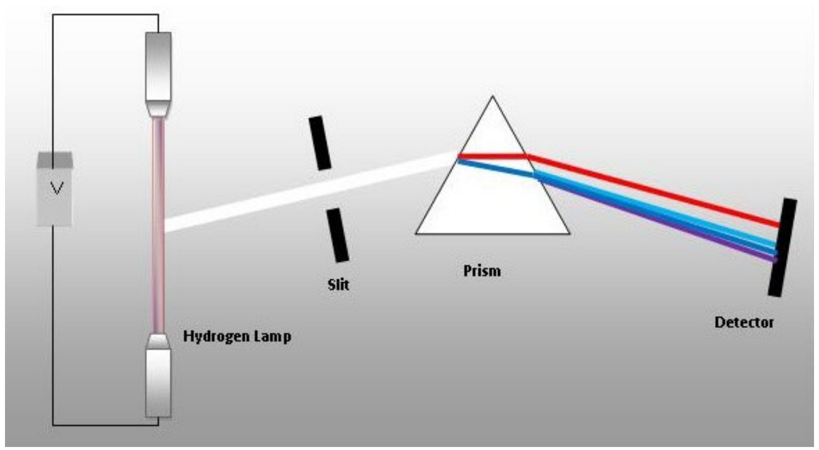
Nature is charge neutral→ need positively charged particles to balance.

- Proton (Thomson + Rutherford, 1907).

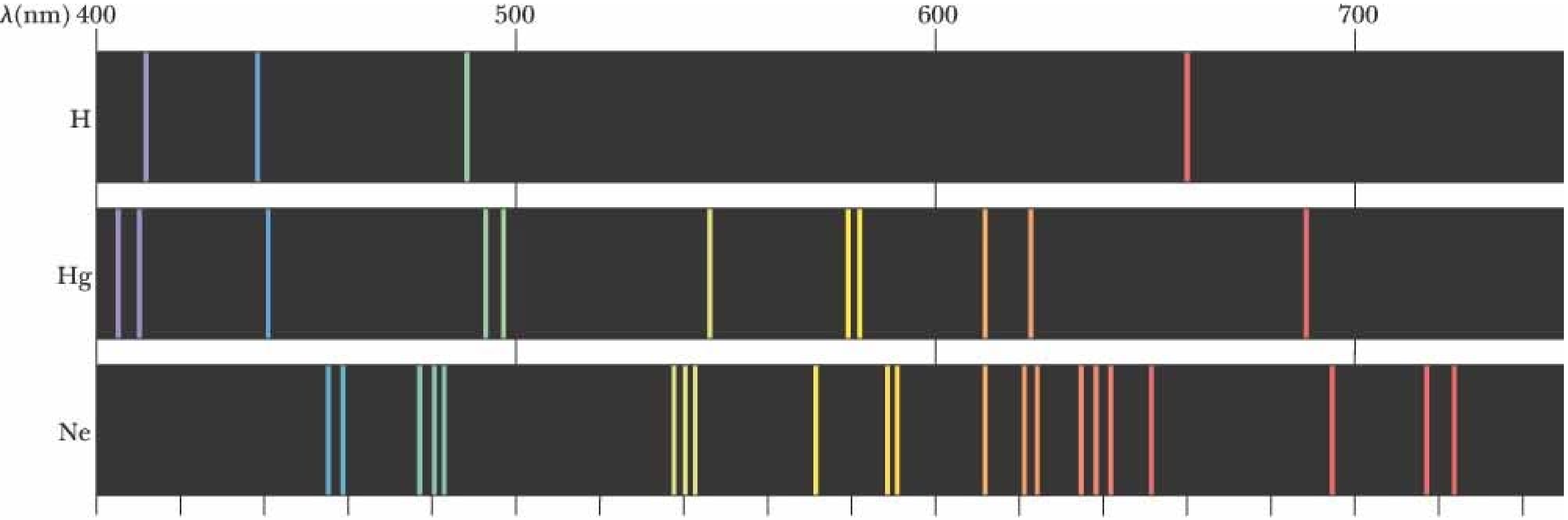


Electrodynamics predicts that the electron spirals into the nucleus within **~10^{-8} s**.

Glimpses into the Atomic Structure



Apply voltage to gas discharge tube, observe emitted light.



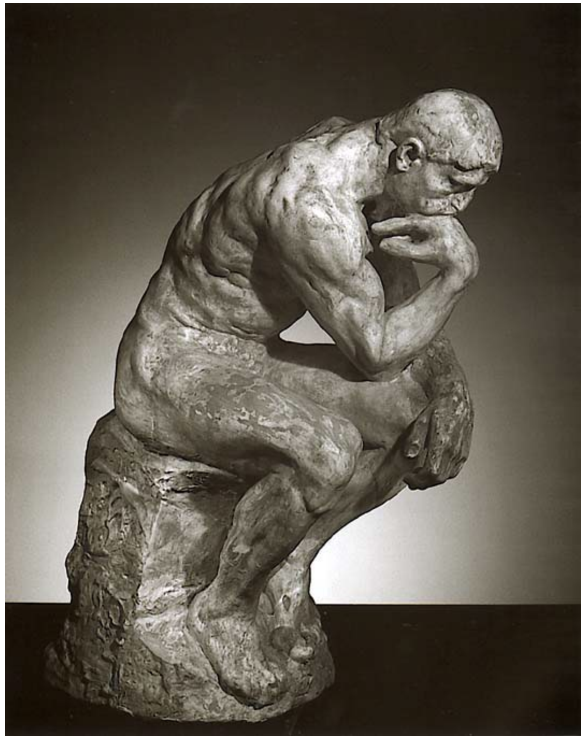
Light emitted from gas occurs at specific (discrete) wavelength.

http://www.physics.umd.edu/courses/Phys401/bedaque07/discrete_spectra.jpg

“Everyday “objects

Classical physics continuous.

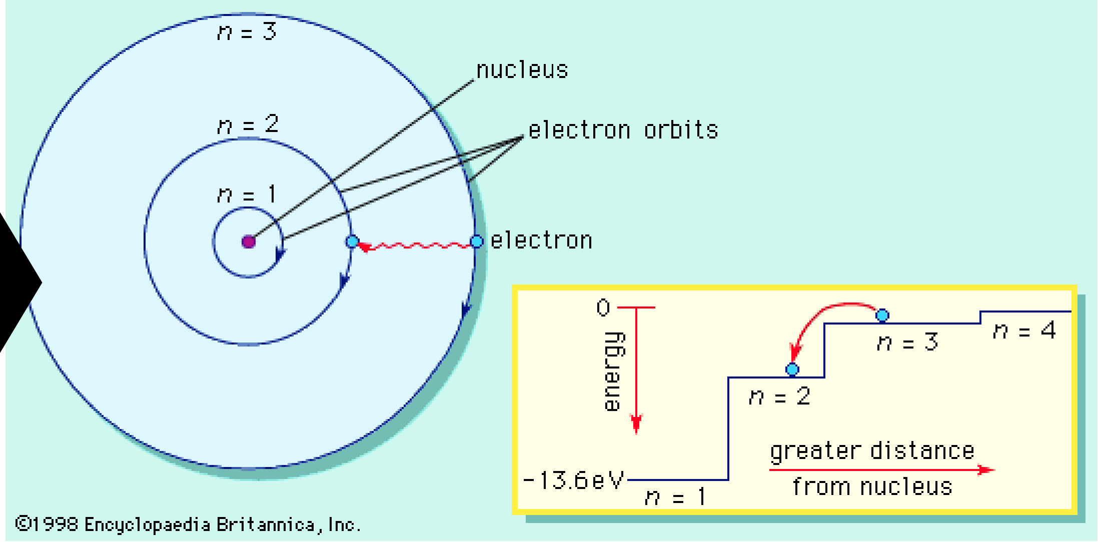
- **Classical Physics Predicts that Matter is Unstable?**
- **Discrete Features?**



Auguste Rodin; The Thinker

Solution: Quantum Mechanics

N. Bohr



N. Bohr (1915)

Hydrogen atom:

$$L = n \cdot \hbar$$

$$E_n = -\frac{13.6 eV}{n^2}$$

$$a_B = n \cdot 0.529 \cdot 10^{-10}\ m$$

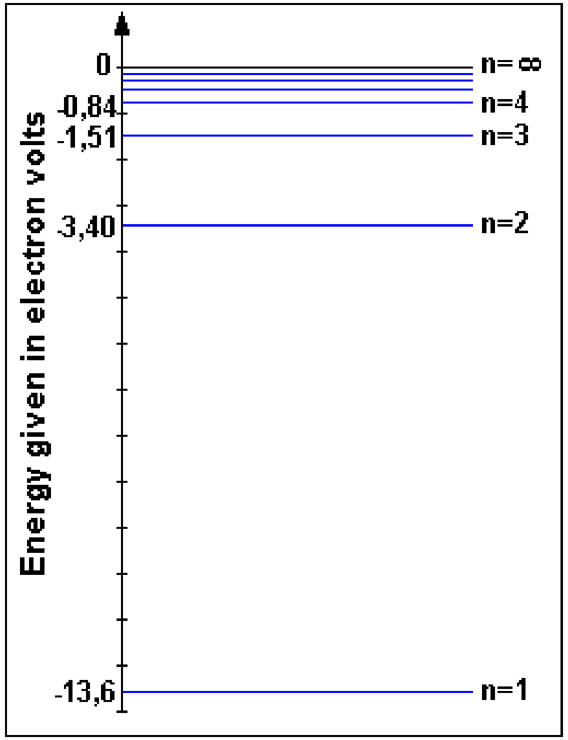
$$= n \cdot 0.529\ A$$

Discrete Spectra

Idea: Emission/adsorption **only allowed** between energy levels.

$$\Delta E = E_f - E_i$$

$$= \frac{91.74\,nm}{\frac{1}{n_f^2} - \frac{1}{n_i^2}}$$



Bohr model predicts:
discrete emission lines.

Absorption lines:

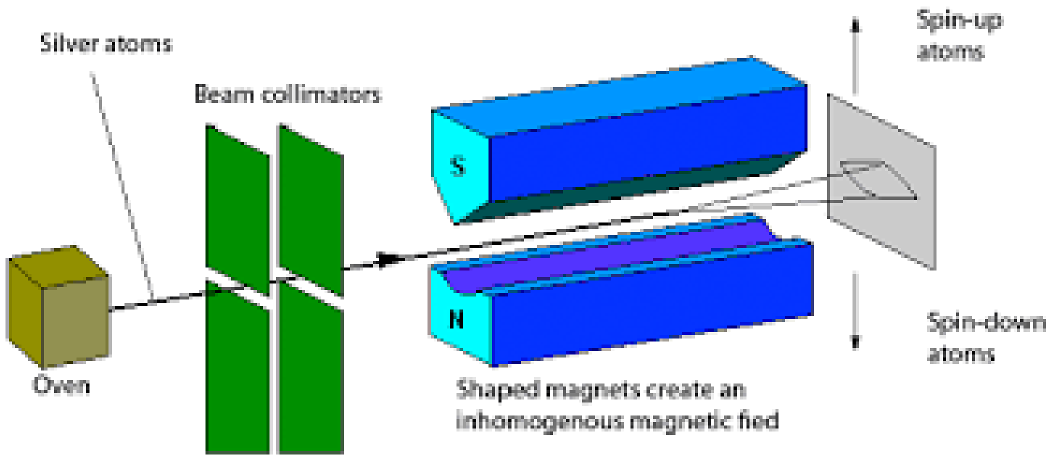
$n_i = 1$ ➔ Lyman series.

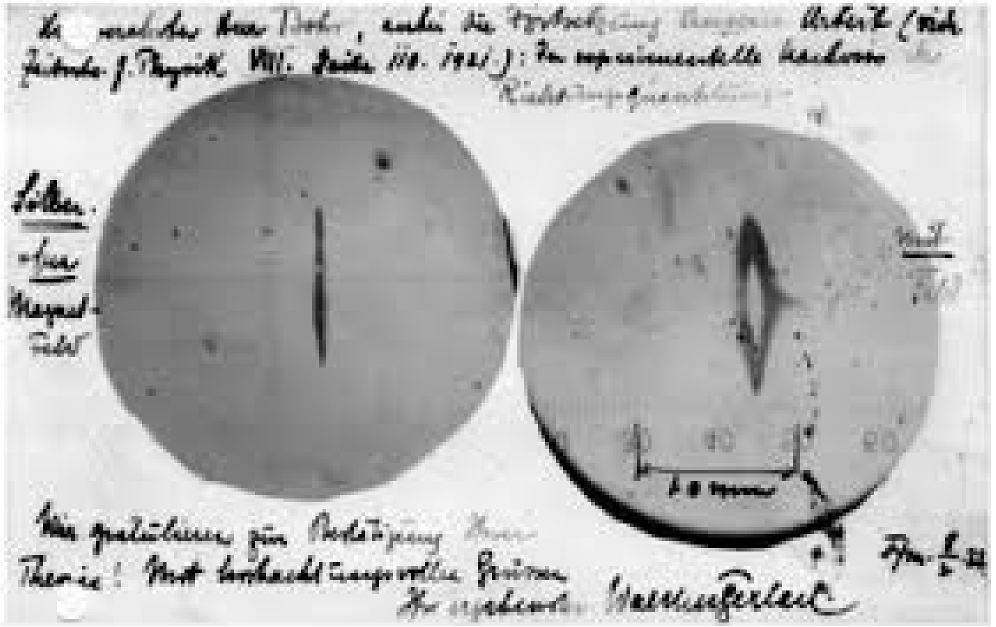
$n_i = 2$ ➔ Balmer series.

$n_i = 3$ ➔ Paschen series.

http://library.thinkquest.org/28383/grafika/1/awodor-poziomy.gif

“A Quantum Discovery: Spin”





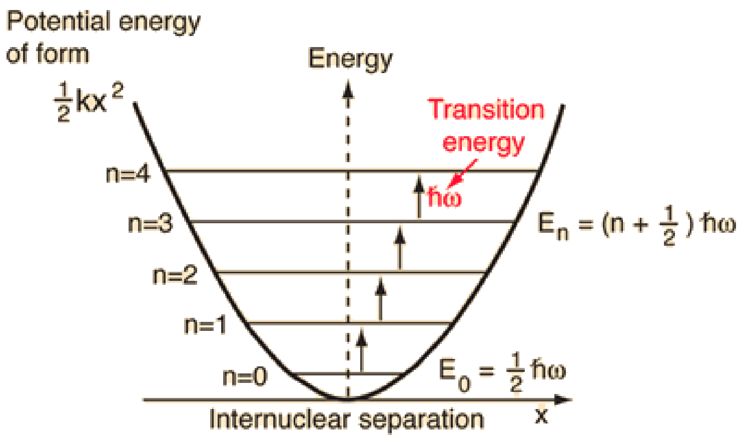
Classical expectation: electron, one magnetic moment. After traversing a strong inhomogeneous magnetic field: all moments are aligned => one spot on a detector.

Stern-Gerlach experiment (1922):
Two spots.

Suggesting two magnetic moment orientation, intrinsic property of the electron.. .

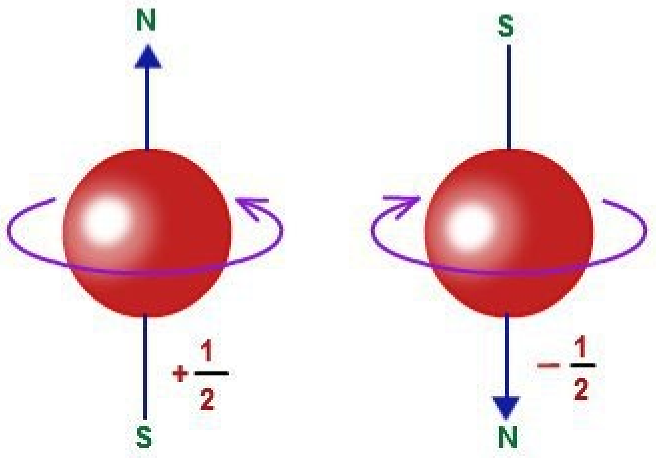
Quantum Mechanical Objects

Quantization (discrete) energy states.



Spin: No classical analog

Electronic, nuclear,…



In 3D, two types of particles:

- spin-1/2: electron, proton…: fermions.

  two identical fermions cannot occupy the same quantum state.
- spin-1: photons,…: bosons.

  any number of bosons can occupy the same quantum state.

Quantum mechanics: theory of matter.

The Periodic Table

Quantum rules:

For a given n (main quantum number == shell):

- we have $l=0,1,\ldots,n-1$ angular momentum states.
- for each l we have $m_l=-l,-(l+1),\ldots,0,(l-1),l$ additional possibilities (z-component of angular momentum).
- Each state can hold up to *2 electrons* (spin).

Example: $n=1$ ➔ $l=0$ ➔ $m_l=0$ ➔ *1 state * 2 spins*

➔ total of *two* possibilities

== 1st row of the periodic table.

In general it can be shown that for a given *n* we can have

$2 * n^2$ possibilities

$n=2$ ➔ *8* possibilities.

(note: 3rd+ row is more complicated)

Periodic Table

I	II											III	IV	V	VI	VII	VIII
H^{1} $1s^1$ $^2S_{1/2}$																	He^{2} $1s^2$ $^1S_0$
Li^{3} $2s^1$ $^2S_{1/2}$	Be^{4} $2s^2$ $^1S_0$											B^{5} $2p^1$ $^2P_{1/2}$	C^{6} $2p^2$ $^3P_0$	N^{7} $2p^3$ $^4S_{3/2}$	O^{8} $2p^4$ $^3P_2$	F^{9} $2p^5$ $^2P_{3/2}$	Ne^{10} $2p^6$ $^1S_0$
Na^{11} $3s^1$ $^2S_{1/2}$	Mg^{12} $3s^2$ $^1S_0$											Al^{13} $3p^1$ $^2P_{1/2}$	Si^{14} $3p^2$ $^3P_0$	P^{15} $3p^3$ $^4S_{3/2}$	S^{16} $3p^4$ $^3P_2$	Cl^{17} $3p^5$ $^2P_{3/2}$	Ar^{18} $3p^6$ $^1S_0$
K^{19} $4s^1$ $^2S_{1/2}$	Ca^{20} $4s^2$ $^1S_0$	Sc^{21} $3d^1$ $^2D_{3/2}$	Ti^{22} $3d^2$ $^3F_2$	V^{23} $3d^3$ $^4F_{3/2}$	Cr^{24} $4s^13d^5$ $^7S_3$	Mn^{25} $3d^5$ $^6S_{5/2}$	Fe^{26} $3d^6$ $^5D_4$	Co^{27} $3d^7$ $^4F_{9/2}$	Nl^{28} $3d^8$ $^3F_4$	Cu^{29} $4s^13d^{10}$ $^2S_{1/2}$	Zn^{30} $3d^{10}$ $^1S_0$	Ga^{31} $4p^1$ $^2P_{1/2}$	Ge^{32} $4p^2$ $^3P_0$	As^{33} $4p^3$ $^4S_{3/2}$	Se^{34} $4p^4$ $^3P_2$	Br^{35} $4p^5$ $^2P_{3/2}$	Kr^{36} $4p^6$ $^1S_0$
Rb^{37} $5s^1$ $^2S_{1/2}$	Sr^{38} $5s^2$ $^1S_0$	Y^{39} $4d^1$ $^2D_{3/2}$	Zr^{40} $4d^2$ $^3F_2$	Nb^{41} $5s^14d^4$ $^6D_{1/2}$	Mo^{42} $5s^14d^5$ $^7S_3$	Tc^{43} $5s^14d^6$ $^6D_{9/2}$	Ru^{44} $5s^14d^7$ $^5F_5$	Rh^{45} $5s^14d^8$ $^4F_{9/2}$	Pd^{46} $5s^04d^{10}$ $^1S_0$	Ag^{47} $5s^14d^{10}$ $^2S_{1/2}$	Cd^{48} $4d^{10}$ $^1S_0$	In^{49} $5p^1$ $^2P_{1/2}$	Sn^{50} $5p^2$ $^3P_0$	Sb^{51} $5p^3$ $^4S_{3/2}$	Te^{52} $5p^4$ $^3P_2$	I^{53} $5p^5$ $^2P_{3/2}$	Xe^{54} $5p^6$ $^1S_0$
Cs^{55} $6s^1$ $^2S_{1/2}$	Ba^{56} $6s^2$ $^1S_0$	57-71 Rare Earths	Hf^{72} $5d^2$ $^3F_2$	Ta^{73} $5d^3$ $^3F_{3/2}$	W^{74} $5d^4$ $^5D_0$	Re^{75} $5d^5$ $^6S_{5/2}$	Os^{76} $5d^6$ $^5D_4$	Ir^{77} $5d^7$ $^4F_{9/2}$	Pt^{78} $6s^15d^9$ $^3D_3$	Au^{79} $6s^15d^{10}$ $^2S_{1/2}$	Hg^{80} $5d^{10}$ $^1S_0$	Tl^{81} $6p^1$ $^2P_{1/2}$	Pb^{82} $6p^2$ $^3P_0$	Bi^{83} $6p^3$ $^4S_{3/2}$	Po^{84} $6p^4$ $^3P_2$	At^{85} $6p^5$ $^2P_{3/2}$	Rn^{86} $6p^6$ $^1S_0$
Fr^{87} $7s^1$ $^2S_{1/2}$	Ra^{88} $7s^2$ $^1S_0$	89-103 Acti- nides	Rf^{104} $5f^{14}6d^2$	Ha^{105}													

Rare earths (Lanthanides)	La^{57} $5d^1$ $^2D_{3/2}$	Ce^{58} $4f^2$ $^3H_4$	Pr^{59} $4f^3$ $^4I_{9/2}$	Nd^{60} $4f^4$ $^5I_4$	Pm^{61} $4f^5$ $^6H_{5/2}$	Sm^{62} $4f^6$ $^7F_0$	Eu^{63} $4f^7$ $^8S_{7/2}$	Gd^{64} $5d^14f^7$ $^9D_2$	Tb^{65} $6s^14f^9$ $^6H_{15/2}$	Dy^{66} $4f^{10}$ $^5I_8$	Ho^{67} $4f^{11}$ $^4I_{15/2}$	Er^{68} $4f^{12}$ $^3H_6$	Tm^{69} $4f^{13}$ $^2F_{7/2}$	Yb^{70} $4f^{14}$ $^1S_0$	Lu^{71} $5d^14f^{14}$ $^2D_{3/2}$
Actinides	Ac^{89} $6d^1$ $^2D_{3/2}$	Th^{90} $6d^2$ $^3F_2$	Pa^{91} $6d^15f^2$ $^4K_{11/2}$	U^{92} $6d^15f^3$ $^5L_6$	Np^{93} $6d^15f^4$ $^6L_{11/2}$	Pu^{94} $5f^6$ $^7F_0$	Am^{95} $5f^7$ $^8S_{7/2}$	Cm^{96} $6d^15f^7$ $^9D_2$	Bk^{97} $6d^15f^8$ $^6G_{15/2}$	Cf^{98} $5f^{10}$ $^5I_8$	Es^{99} $5f^{11}$ $^4I_{15/2}$	Fm^{100} $5f^{12}$ $^3H_6$	Md^{101} $5f^{13}$ $^2F_{7/2}$	No^{102} $5f^{14}$ $^1S_0$	Lr^{103} $5f^{14}6d^1$ $^2D_{3/2}$

FIGURE 12.5

Quantum mechanics: theory of matter.

Quantum Information Science (QIS)

Dr. Boris Kiefer, Lecture 2

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Quantum Communication
Quantum Sensing

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- # Quantum States.

A quantum state is a mathematical representation of a physical system, such as an atom, and provides the basis for processing quantum information.

1. Quantum states are represented by vectors in an abstract space,

$$|0>, |1>$$

$$|\Psi>= a_0|0> +a_1|1>;a_0^2 + a_1^2 = 1$$

2. The direction of a quantum state vector determines the probabilities of all possible outcomes of a measurement. This captures a behavior that cannot solely be captured by the arthimetic of probability.

$$|\Psi>= |a_0|0> +a_1|1>;a_0^2 + a_1^2 = 1$$

• Quantum States.

3. Quantum systems are fragile. For instance, measurement almost always disturbs a quantum system in a way that cannot be ignored. This fragility influences the design of computational algorithms, communication, and sensing protocols. For example, the orientation of the state vector before and after measurement may differ: projection erases any non-parallel components to the state vector after measurement. May be one of the most succinct expressions of this statement is the Heisenberg uncertainty principle, for applied to position and linear momentum ($p = mv$):

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Therefore, the smaller the desired uncertainty in location (smaller Δx), the larger the corresponding uncertainty in momentum (Δp, direction and magnitude).

- # Measurements.

Quantum applications are designed to carefully manipulate fragile quantum systems without observation to increase the probability htat the final measurement will provide the intended result.

1. A measuremetn is an interaction with the quantum system that transforms a state with multiple possible outcomes into a "collapsed" state that now has only one outcome: the measured outcome.
2. A quantum state determines the probability of the outcome of a single quantum measurement, but one outcome rarely reveals complete information of the system.
3. Repeated measurements on identically prepared quantum systems are required to determin more complete information about the (quantum) state.
4. Because of the limitations of quantum measurements (providing only partial information and disturbing the sysyem), quantum states cannot be copies or duplicated.

- **Measurement.**

$$|\Psi >= |a_0|0 > +a_1|1 >; a_0^2 + a_1^2 = 1$$

and the probability to observe the system in one of the two possible states is:

$$|0 >: |a_0|^2$$
$$|1 >: |a_1|^2$$

$$M_0 = |0 >< 0|$$
$$Pr[|0 >] = |M_0|\Psi > |^2 =< \Psi|M_0^\dagger M_0|\Psi >$$

with results in the new quantum state:

$$|\Psi' >= \frac{M_0|\Psi >}{\sqrt{< \Psi|M_0^\dagger M_0|\Psi >}}$$

- # Qubits.

The qubit is the fundamental unit of quantum information, and is encoded in a physical system, such as polarization states of light, energy states of an atom, or spin states of an electron.

1. Unlike a classical bit, a qubit represents information in a superposition, or vector sum that incorporates two mutually exclusive quantum states.

2. At a particular moment in time a, a set of N classical bits can only exist in N^{I} possible states, but a set of N qubits can exist in a superposition of all 2^N these states. This capability allows quantum information to be stored and processed in ways that would be difficult or impossible to do classically.

3. Multiple qubits can be entangled, where the measurement outcome of one qubit is correlated with the measurement outcomes of the others.

- **Qubits.**

Example: 3 Qubit States:

$$\begin{pmatrix}0\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix},\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\1\\1\end{pmatrix}$$

In the quantum mechanical version, linear combination of these states are allowed:

$$a_0\begin{pmatrix}0\\0\\0\end{pmatrix}+a_1\begin{pmatrix}0\\0\\1\end{pmatrix}+a_2\begin{pmatrix}0\\1\\0\end{pmatrix}+a_3\begin{pmatrix}0\\1\\1\end{pmatrix}+a_4\begin{pmatrix}1\\0\\0\end{pmatrix}+a_5\begin{pmatrix}1\\0\\1\end{pmatrix}+a_6\begin{pmatrix}1\\1\\0\end{pmatrix}+a\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\sum |a_i|^2 = 1$$

- # Entanglement

Entanglement, an inseperable relationship between multiple qubits, is a key property of quantum systems necessary for obtaining a quantum advantage in most QIS applications.

1. When multiple quantum system in superposition are entangled, their measuremnts outcomes are correlated. Entanglement can cause correlations that are different from what is possible in classical systems.

2. An entangled quantum system of multipl equbits cannot be described solely by specifying a an individual quantum state for each qubit.

3. Quantum technologies rely on entanglement in different ways. When a fragile entangled state is maintained, a computational advantage can be realized. The extreme sensitivity of entangled states, however, can enhance sensing and communication.

• Entanglement: Bell States

$$|\Phi^+ >= \frac{1}{\sqrt{2}}(|00 > +|11 >)$$

Let's try to write this as a tensor product of two single qubits:

$$|a >= a_0|0 > +a_1|1 >$$
$$|b >= b_0|0 > +b_1|1 >$$
$$|a > \otimes |b >= a_0 b_0|00 > +a_1 b_0|10 > +a_0 b_1|01 > +a_1 b_1|11 >$$

and we compare the coefficients, one-by-one:

$$a_0 b_0 = \frac{1}{\sqrt{2}}$$
$$a_1 b_0 = 0$$
$$a_0 b_1 = 0$$
$$a_1 b_1 = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ **All amplitudes are zero**
$\Rightarrow$ **entangled.**

- # Entanglement: Bell States

$$|\Phi^+ >= \frac{1}{\sqrt{2}}(|00> + |11>)$$

$$|\Phi^- >= \frac{1}{\sqrt{2}}(|00> - |11>)$$

$$|\Psi^+ >= \frac{1}{\sqrt{2}}(|01> + |10>)$$

$$|\Psi^- >= \frac{1}{\sqrt{2}}(|01> - |10>)$$

Entangled States realize state correlations between quantum objects, regardless of distance: Teleportation,…

Therefore, the question arises how to generate an entangled state?

• Entanglement: Creating a Bell State

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Input	Output
$\|0\rangle$	$\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$
$\|1\rangle$	$\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$

Therefore, the question arises how to generate an entangled state? Let's assume

that we can initialize a unique state, say $|0>$. First we generate a superposition state by applying a Hadamard gate to this state:

$$|s>= H|0>$$

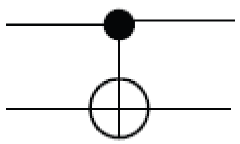
$$= \frac{1}{\sqrt{2}}(|0><0| + |1><0| + |0><1| + |1><1|)|0>$$

$$\frac{1}{\sqrt{2}}(|0> + |1>)$$

or in matrix notation:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- # Entanglement: Creating a Bell State



$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

now we have to apply a "disruptive" step that entangles this state with a well-defined second qubit, that we assume to be in state $|0>$. So, the current state of the system is:

$$\frac{1}{\sqrt{2}}(|0> + |1>) \otimes |0> = \frac{1}{\sqrt{2}}(|00> + |10>)$$

Since this state was generated as a tensor product, it is clearly not entangled.

How does this state transform if we apply a CNOT operation:

$$\begin{aligned} X &= |1>< 0| + |0>< 1| \\ CNOT &= |0>< 0| \otimes I + |1>< 1| \otimes X \\ &= |0>< 0| \otimes (0>< 0| + |1>< 1|) + |1>< 1| \otimes (1>< 0| + |0>< 1|) \\ &= |00>< 00| + |01>< 01| + |11>< 10| + |10>< 11| \end{aligned}$$

and the action of CNOT can easily be read off from the last equation:

Entanglement: Creating a Bell State

and the action of CNOT can easily be read off from the last equation:

$$|00> \rightarrow |00>$$
$$|01> \rightarrow |01>$$
$$|10> \rightarrow |11>$$
$$|11> \rightarrow |10>$$

and the matrix representation is of CNOT is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

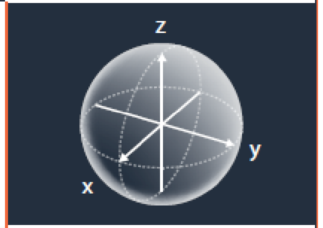
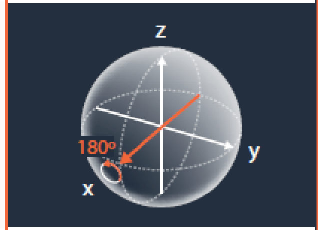
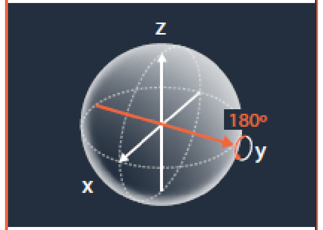
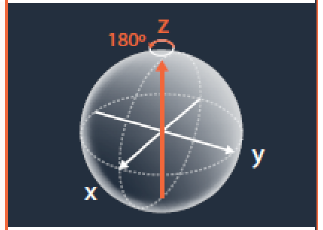
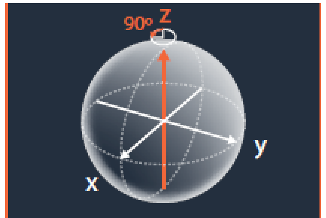
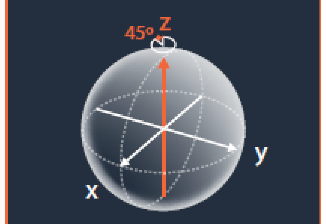
Now, we can apply CNOT to our superposition state:

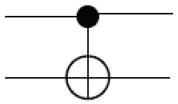
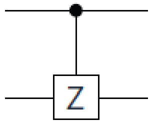
$$CNOT\left(\frac{1}{\sqrt{2}}(|00> + |10>)\right)$$
$$= \frac{1}{\sqrt{2}}(|00> + |11>) = |\Phi^+>$$

Entangled State = CNOT * Hadamard * |0>

(Standard procedure after initialization of qubits to create entangled states).

• Quantum Computers – Gates.

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE	BLOCH SPHERE
I Identity-gate: no rotation is performed.	—[I]—	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|0\rangle$; $\|1\rangle$ → $\|1\rangle$	
X gate: rotates the qubit state by π radians (180º) about the x-axis.	—[X]—	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|1\rangle$; $\|1\rangle$ → $\|0\rangle$	
Y gate: rotates the qubit state by π radians (180º) about the y-axis.	—[Y]—	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $i\|1\rangle$; $\|1\rangle$ → $-i\|0\rangle$	
Z gate: rotates the qubit state by π radians (180º) about the z-axis.	—[Z]—	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|0\rangle$; $\|1\rangle$ → $-\|1\rangle$	
S gate: rotates the qubit state by $\frac{\pi}{2}$ radians (90º) about the z-axis.	—[S]—	$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|0\rangle$; $\|1\rangle$ → $e^{i\frac{\pi}{2}}\|1\rangle$	
T gate: rotates the qubit state by $\frac{\pi}{4}$ radians (45º) about the z-axis.	—[T]—	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|0\rangle$; $\|1\rangle$ → $e^{i\frac{\pi}{4}}\|1\rangle$	
H gate: rotates the qubit state by π radians (180º) about an axis diagonal in the x-z plane. This is equivalent to an X-gate followed by a $\frac{\pi}{2}$ rotation about the y-axis.	—[H]—	$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$; $\|1\rangle$ → $\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$	

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE
Controlled-NOT gate: apply an X-gate to the target qubit if the control qubit is in state $\|1\rangle$		$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	Input / Output: $\|00\rangle$ → $\|00\rangle$; $\|01\rangle$ → $\|01\rangle$; $\|10\rangle$ → $\|11\rangle$; $\|11\rangle$ → $\|10\rangle$
Controlled-phase gate: apply a Z-gate to the target qubit if the control qubit is in state $\|1\rangle$		$CPHASE = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	Input / Output: $\|00\rangle$ → $\|00\rangle$; $\|01\rangle$ → $\|01\rangle$; $\|10\rangle$ → $\|10\rangle$; $\|11\rangle$ → $-\|11\rangle$

Krantz et al. (2019)

- **Decoherence.**

For quantum information applicaitons to be successfully completed, fragile quantum states must be preserved, or kept coherent.

1. Decoherence erodes superposition and entanglement of undesired interacton with the surrounding environment. Uncontrolled radiation, inclusing light, vibration, heat, or magnetic fields, can all cause decoherence.

2. Some types of qubits are inherently isolated, whereas others require carefully engineered materials to maintain their coherence.

3. High decoherence rates limit the length and complexity of quantum computations; implementing methods that corect errors can mitigate these errors.

- # Quantum Computers.

Quantum computers, which use qubits and quantum operations, will solve certain complex computational problems for efficiently than calssical computers.

1. Qubits can represent information compactly; more information can be stored and processed using 100 qubits than the largest conceivable classical supercomputer.

2. Quantum data can be kept in a superpposition of exponentially many classical states during processing, giving quantum computers a significant speed advantage for certain computations such as factoring large numbers (exponential speed-up) and performing searches (quadratic speed-up). However, there is no speed advantage for many other types of computations.

3. A fault tolerant quantum computer corrects all errors that occur during quantum computation, inclusing those arising from decoherence, but error correction requires significantly more resources than the original computation.

Quantum Communication

- **Quantum Communication.**

Quantum communication uses entanglement or a tranmission channel, such as a optical fiber, to transfer quantum information betwen different locations.

1. Quantum teleprotation is a protocol that uses entanglement to destroy quantum information at one location and and recreate it at a second site, without transferring physical qubits.

2. Quantum cryptography enhances provacy based on quantum physical principles and cannot be circumvented. Due to the fragility of quantum systems, an eavesdropper's interloping measurement will almost always be detected.

Quantum Sensing – A Primer

Quantum Sensing – Introduction

Quantum sensing uses quantum states to detect and measure physical properties with the highest precision allowed by quantum mechanics.

1. The Heisenberg uncertainty principle describes a fundamental limit in simultaneous measureing two specific, separate attributes. “Squeezing” deliberatly sacrifices the certainty of measuring one attribute in order to achieve higher precision in measureing the other attribute; for example squeezing is used in LIGO to improve the sensitivity to gravitational waves.

2. Quantum sensors take advantage of the fact that physical qubits are extremely sensitive their surroundings. The same fragility that leads to rapid decoherence enables precise sensors. Examples include magnetometers, single-photon detectors, and atomic clocks for improvement of medical imaging, navigation, position, and timing.

3. Quantum sensing has vastly improved the precision and accuracy of measuremnts of fundamental constants, freeing the International System of Units from its dependence on one-of-a-kind artifacts. Measurement units are now defined through these fundamental constants, like the speed of light and Planck’s constant.

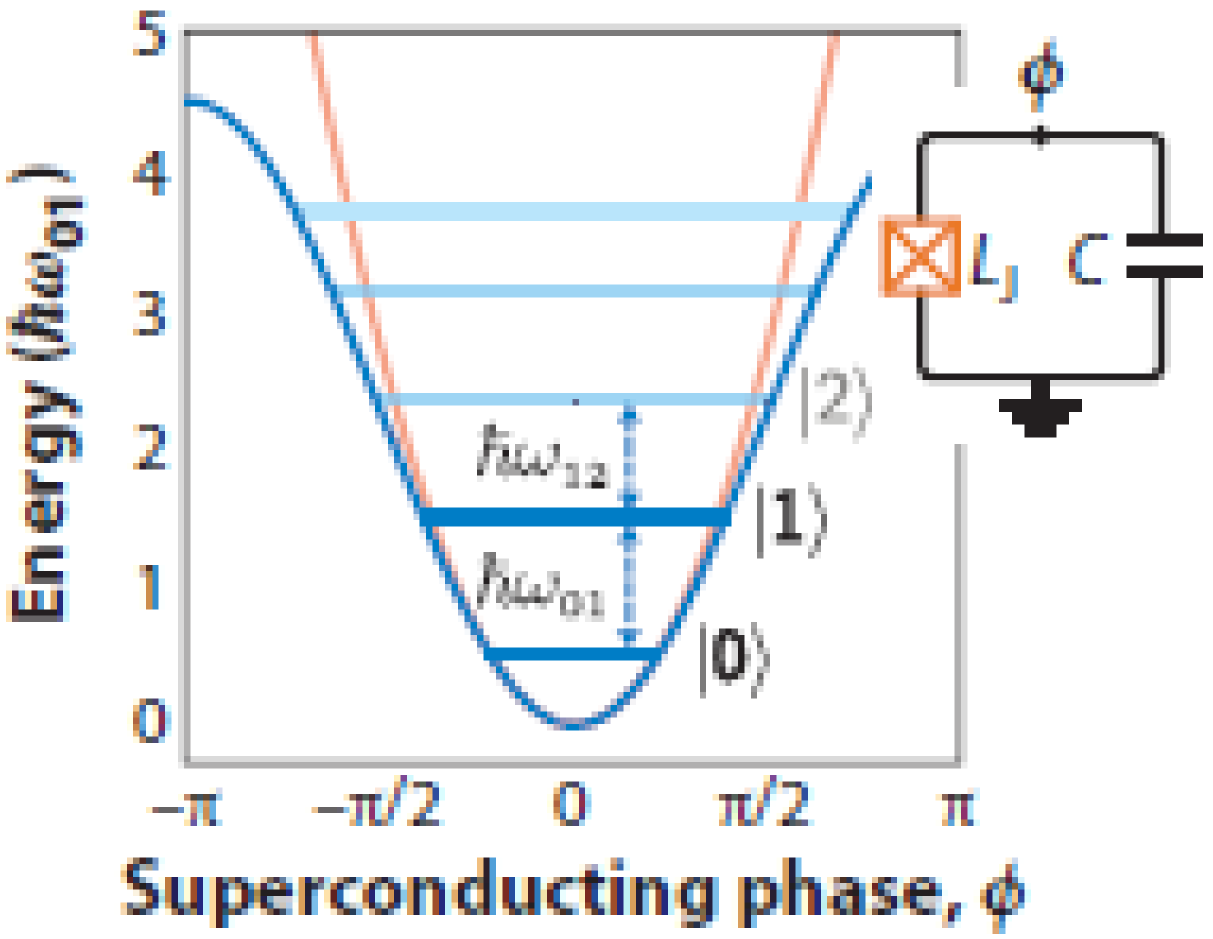
Quantum Sensing

If energy levels are nearly equally spaced:

⇒ Initializing qubits is difficult/impossible.

Flipside:

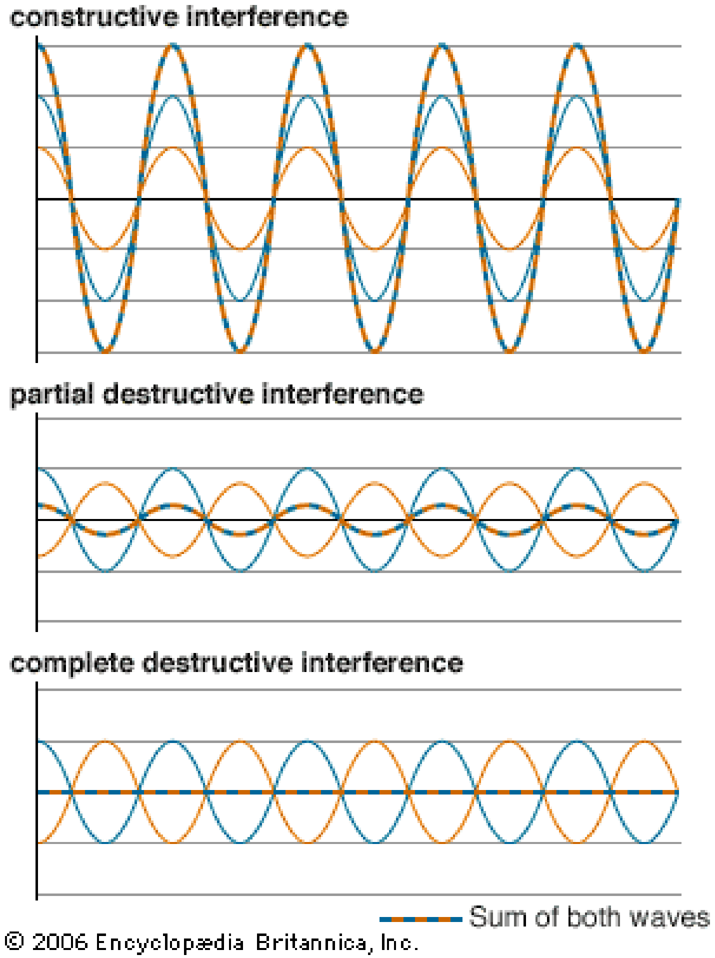
Weak interactions can be resolved ➔ excellent sensor for weak interactions/signals.



Quantum Sensing – Interference

Waves can interfere

- Amplify.
- Weaken.



https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0/section/11.5/primary/lesson/wave-interference-ms-ps

Quantum Sensing – Matter Waves

De Broglie (1924)

The Nature of Light?

Newton: Particle.

Young: Wave.

Einstein (photoelectric effect): Particle.

https://www.metmuseum.org/art/collection/search/191811

https://www.aps.org/publications/apsnews/201010/physicshistory.cfm

Particle:

$$E = mc^2$$

$$E = pc$$

Wave:

$$c = \lambda f$$

$$E = hf$$

$$E = hc/\lambda$$

Combine:

$$\boldsymbol{\lambda = h/p}$$

Quantum Sensing – Matter Waves

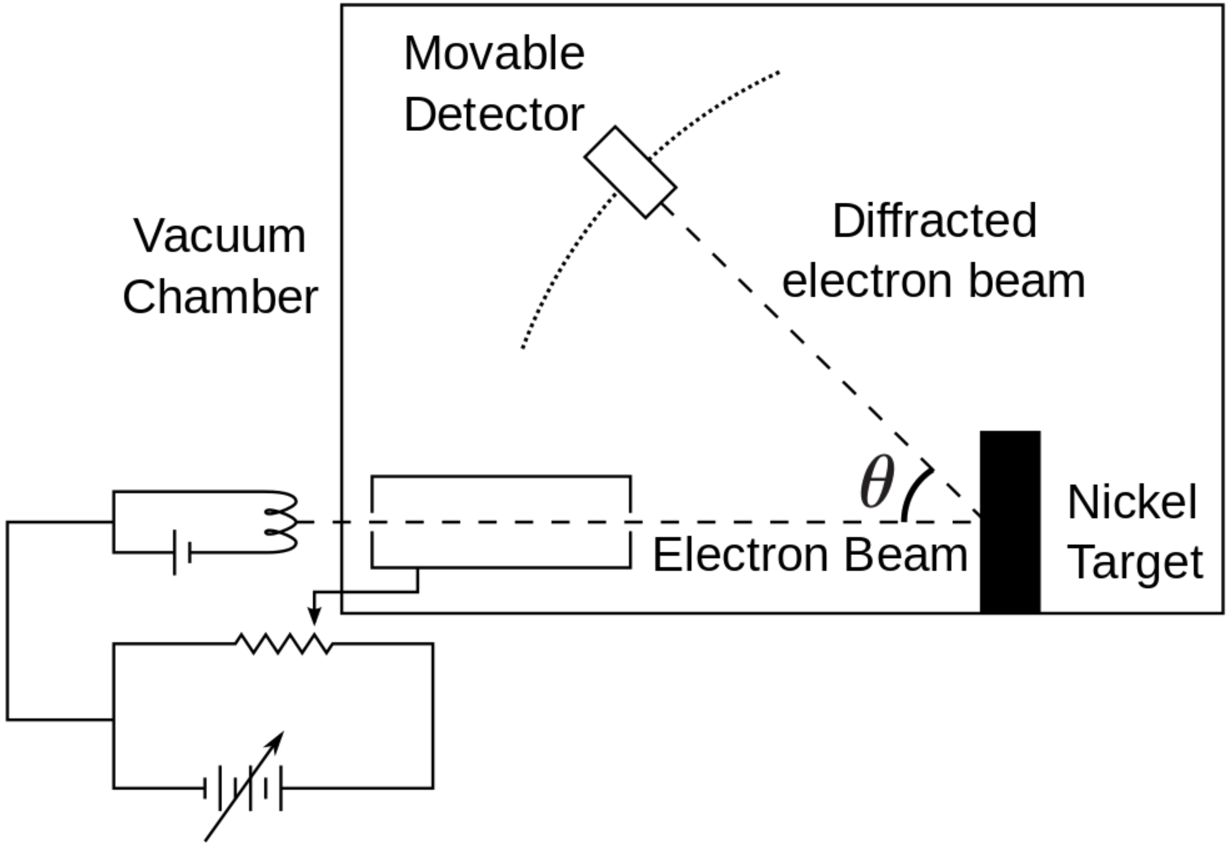
Davisson and Germer (1925)

Electron beam on Ni target.
Objective:

Study angular distribution of electrons emitted from the Ni target.

Vacuum failure ➔ oxide formation ➔ remove oxide at high temperatures.

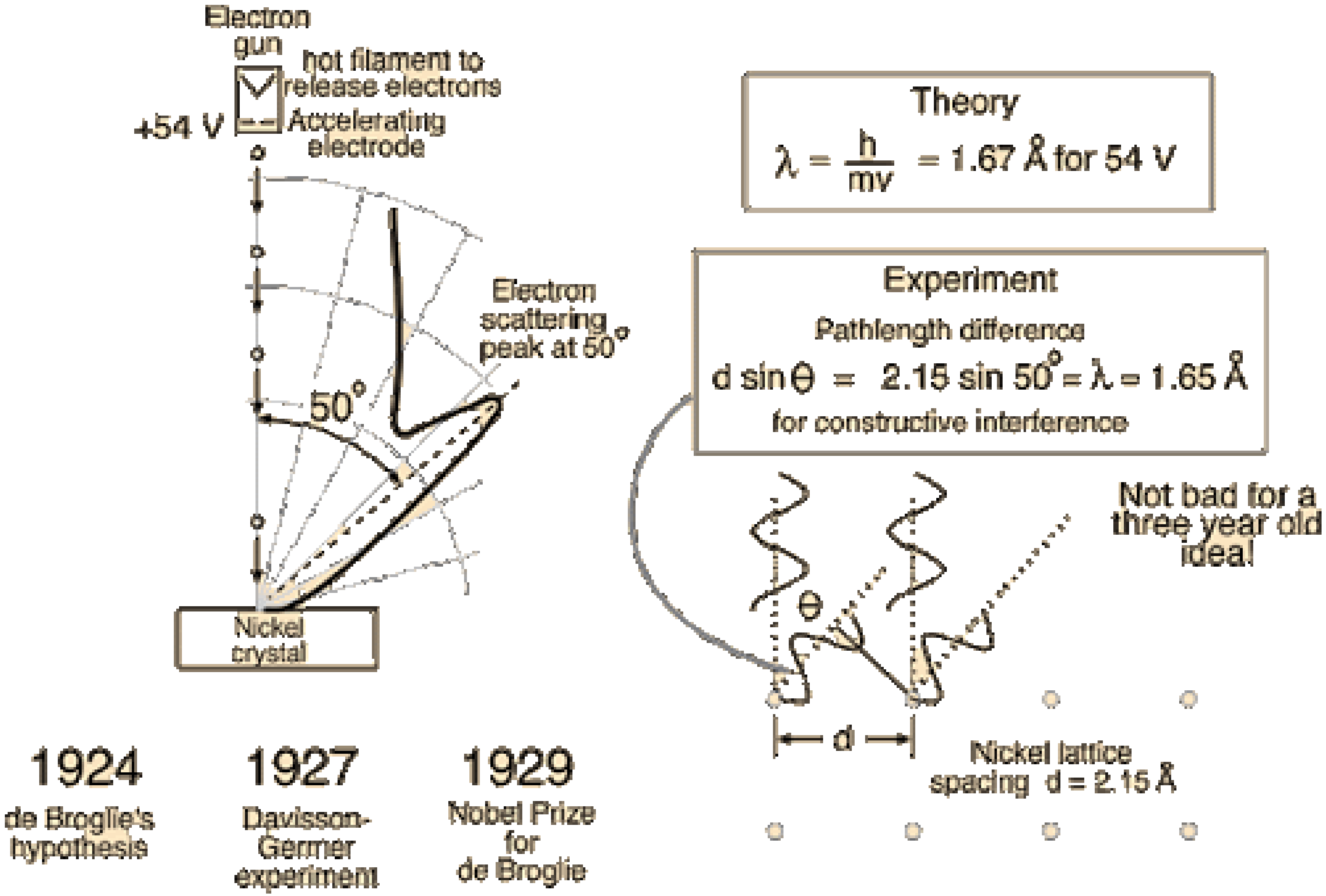
Unintentional: Ni single crystal.



https://en.wikipedia.org/wiki/Davisson%E2%80%93Germer_experiment

Quantum Sensing – Matter Waves

Davisson and Germer (1925)



De Broglie:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$E = \frac{mv^2}{2}$$

$$E = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = 1.67\ \text{Å}$$

Electron diffraction

http://hyperphysics.phy-astr.gsu.edu/hbase/DavGer.html

Quantum Sensing – Gravity

First gravity measurement:
Neutrons from nuclear reactor (immobile)

Neutrons are particles:
Split into two beams.

A-C: slowing down, C-D, slow.
C-D: longer wavelength
A-B: fast, B-D slowing down.
B-D: shorter wavelength

=> Interference pattern CHANGES.

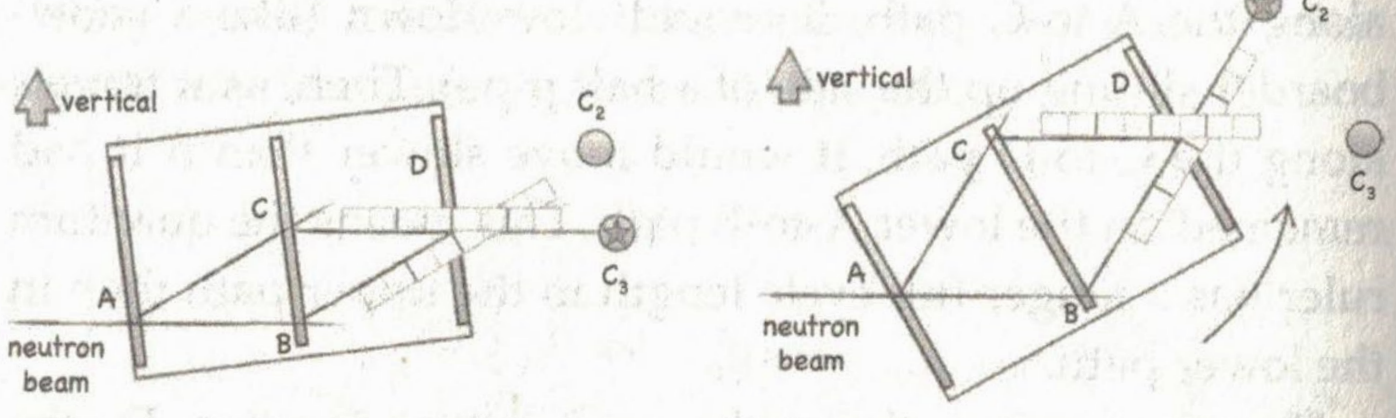
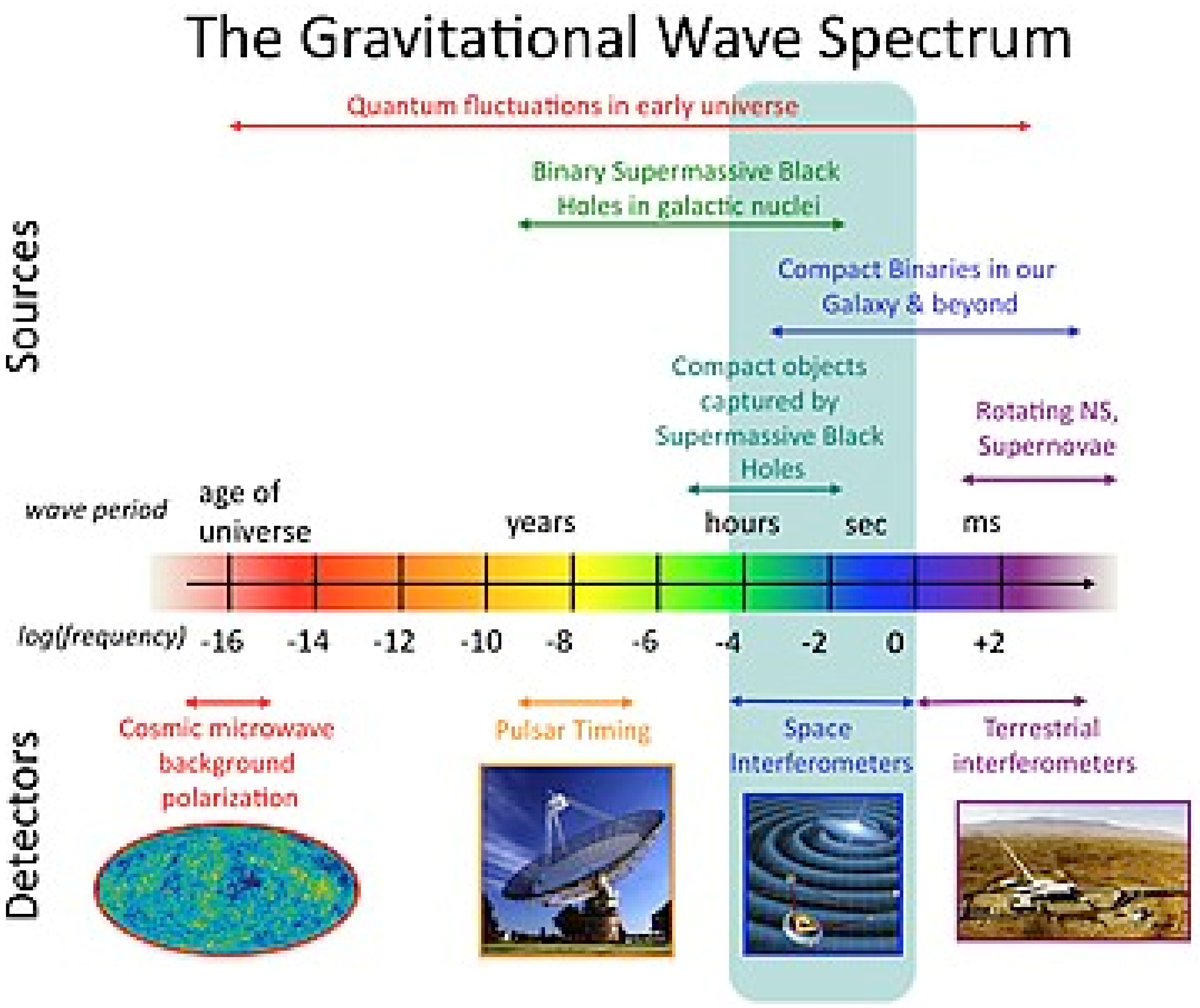


Figure 5.2 When the neutron interferometer is tilted so the upper path is raised higher, the effect of gravity influences the interference and causes the emerging neutrons to switch from detector C_3 to detector C_2.

Alternating constructive/destructive interference: Repetition (angle) increment depends on local gravity.

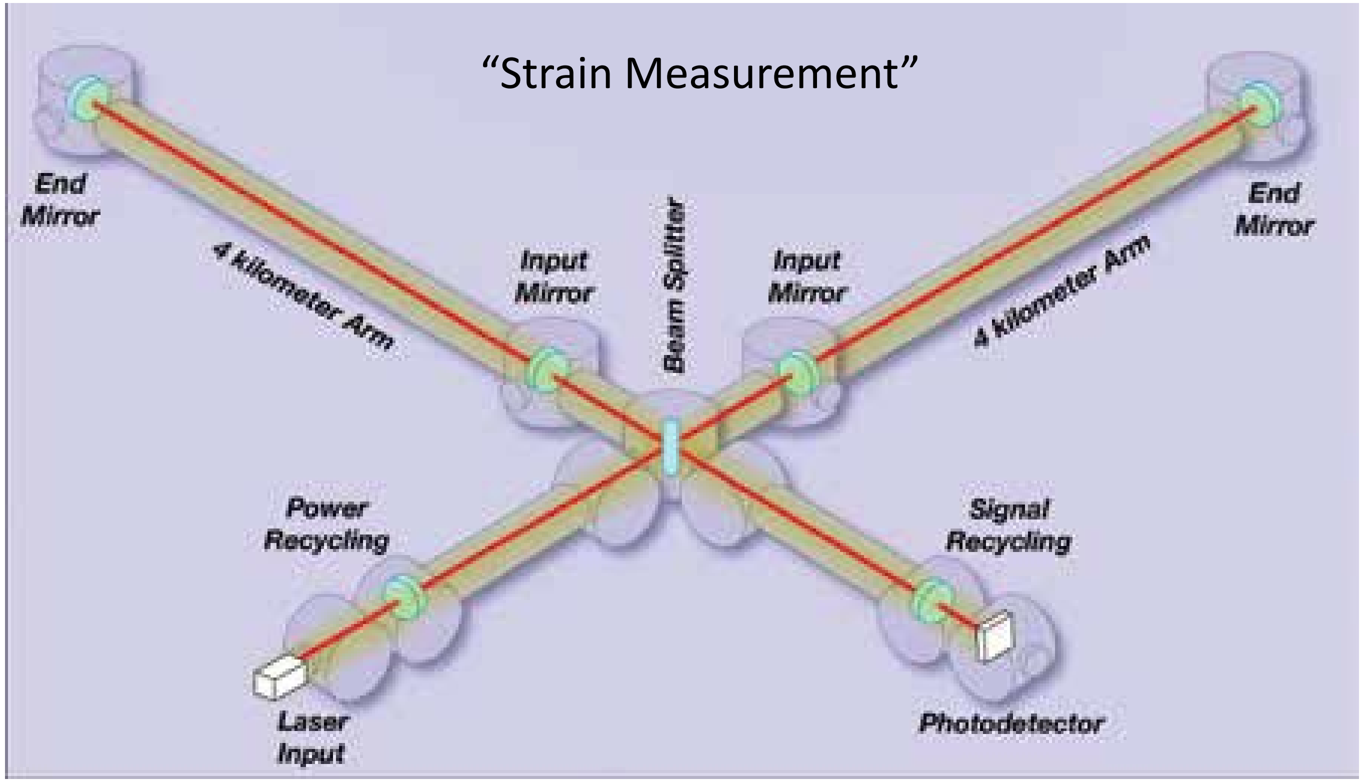
Raymer, M. G., "Quantum Physics" (2017)

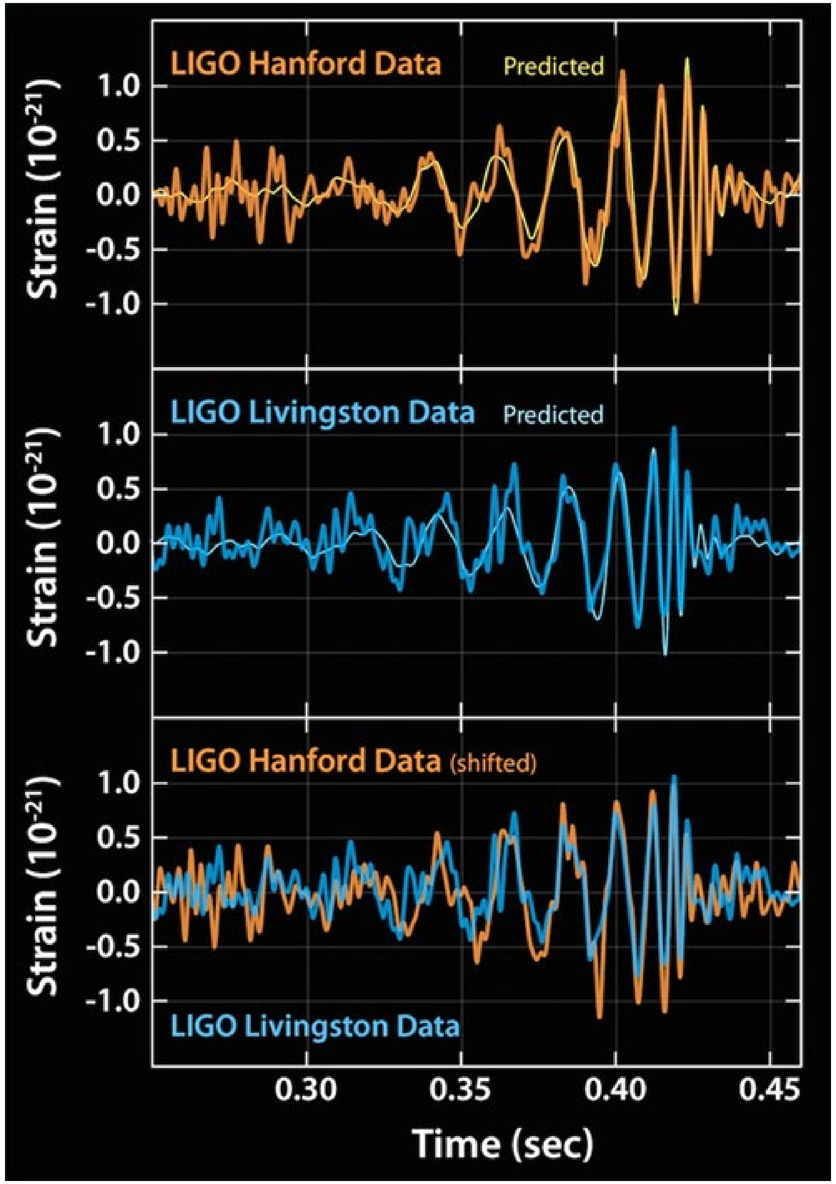
Quantum Sensing – Gravity



https://en.wikipedia.org/wiki/Gravitational_wave

Quantum Sensing – Gravitational Waves, LIGO





Hanford: 0.007s delayed, distance = 2000km

⇒speed: v=distance/time =285714 km/s.

Within errorbar consistent with speed of light

⇒ Gravitational waves travel at speed of light

⇒ Gravitational waves are massless.

http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/gravwav.html#c1

Quantum Sensing – Gravitational Waves, LIGO

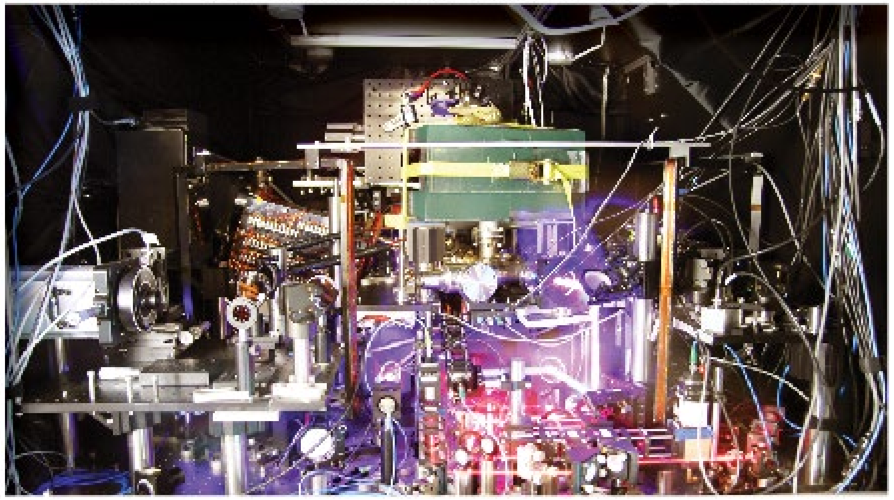
Falling objects (kinematics):

$$t = \sqrt{2gh}$$

Observation:
Time depends on gravity
=> changes in gravity can be measured using sensitive clocks.

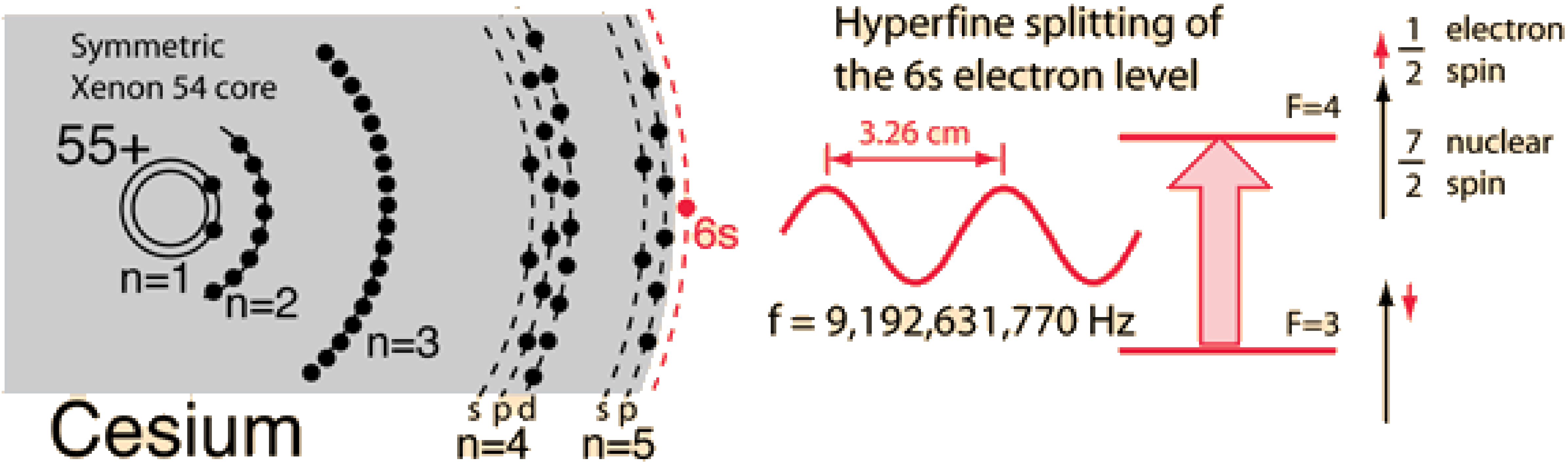
Satellite:
Gravitational wave passes.
Oscillatory change in gravity.

1 NIST-F2 cesium fountain atomic clock



https://www.pacw.org/nist-u-s-time-standard-nist-f2-atomic-clock

Quantum Sensing – Atomic Clocks



All atoms have identical energy levels

$\Rightarrow$ Energy for transition between energy levels are identical

$\Rightarrow$ Frequency of emitted/absorbed photons is identical.

$\Rightarrow$ Ideal clocks/time keepers.

Quantum Sensing – Gravitational Waves

Satellite:
Gravitational wave passes.
Oscillatory change in gravity.

Noise?!?
Reduce by using three satellites.

https://www.google.com/search?q=satellites+quantum+kasevitch&tbm=isch&ved=2ahUKEwiXm9_IhoPvAhUBcs0KHYrSCTUQ2-cCegQIABAA&oq=satellites+quantum+kasevitch&gs_lcp=CgNpbWcQA1CvggFYpJoBYOGbAWgAcAB4AIAB2wGIAfENkgEFMC45LjGYAQCgAQGqAQtnd3Mtd2l6LWltZ8ABAQ&sclient=img&ei=h402YNe7OoHktQaKpaeoAw&bih=667&biw=13
&rlz=1C1CHBF_enUS811US811#imgrc=pRQ_nv1UzBOqzM

Chapter 13.7: (The Physics and Engineering Behind) Building Quantum Computers

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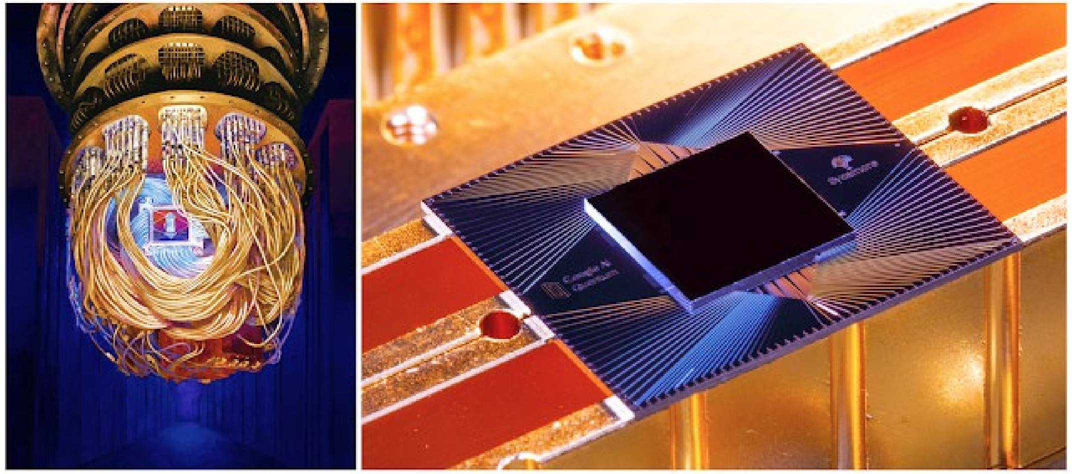
From Concept to (Quantum)Computer

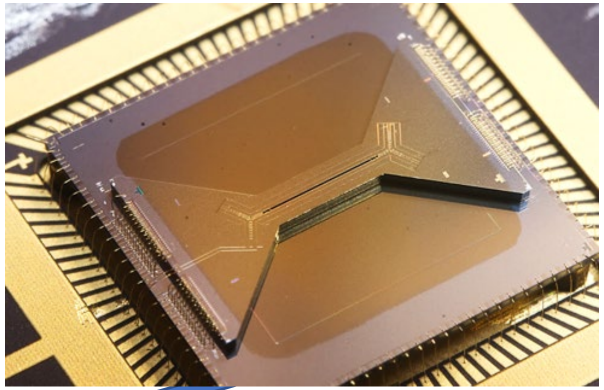
From Concept to (Quantum)Computer

$$H|\psi > E|\psi >$$

$$|\psi(x,t) >= e^{-\frac{i\dot{H}t}{\hbar}} |\psi(x,0) >$$



Google: Sycamore processor



SNL: ion trap

https://ai.googleblog.com/2019/10/quantum-supremacy-using-programmable.html

https://www.sandia.gov/quantum/

DiVincenzo criteria for qubit design (DiVincenzo, 2000):

- Scalable system with well-characterized qubits.
- Ability to initialize qubits.
- Stability of qubits.
- Support of universal computation.
- Ability to measure qubits.

• Quantum Computers – Universal Gate Sets

A common universal quantum gate set is

$$\mathcal{G}_0 = \{\mathsf{X}_\theta, \mathsf{Y}_\theta, \mathsf{Z}_\theta, \mathrm{Ph}_\theta, \mathsf{CNOT}\} \tag{71}$$

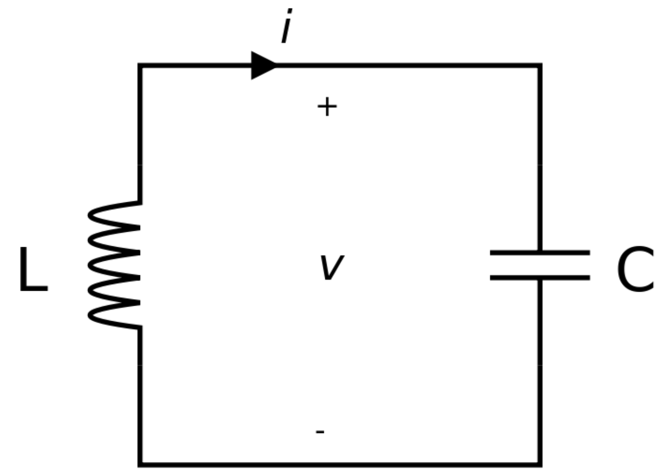
where $\mathrm{Ph}_\theta = e^{i\theta}\mathbb{1}$ applies an overall phase θ to a single qubit. For completeness we mention another universal gate set which is of particular interest from a theoretical perspective, namely

$$\mathcal{G}_1 = \{\mathsf{H}, \mathsf{S}, \mathsf{T}, \mathsf{CNOT}\}, \tag{72}$$

Krantz et al. (2019)

Quantized – LC Circuit

Low temperature



$$H = \frac{\phi^2}{2L} + \frac{1}{2}L\omega^2 Q^2$$

where Q is the charge operator, and ϕ is the
represents the energy stored in a capacitor.
independent Schrödinger equation,

$$H|\psi\rangle = E|\psi\rangle$$

$$E\psi = -\frac{\hbar^2}{2L}\nabla^2\psi + \frac{1}{2}L\omega^2 Q^2\psi$$

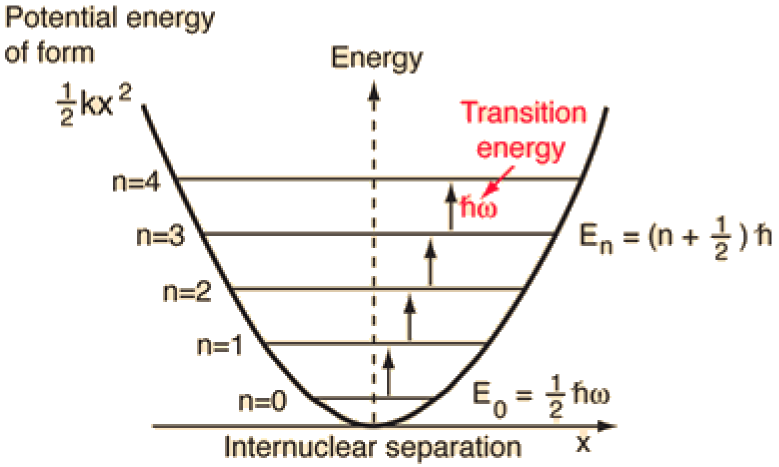
$$\phi \rightarrow \hat{\phi}$$

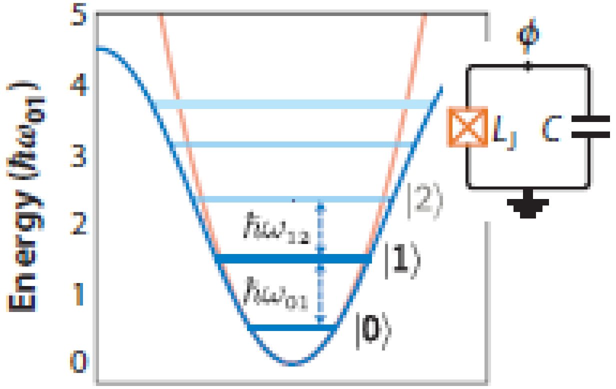
$$q \rightarrow \hat{q}$$

$$H \rightarrow \hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C}$$

and enforcing the canonical commutation relation

$$\left[\hat{\phi}, \hat{q}\right] = i\hbar$$





Creating Anharmonicity
Superconductors – Foundations

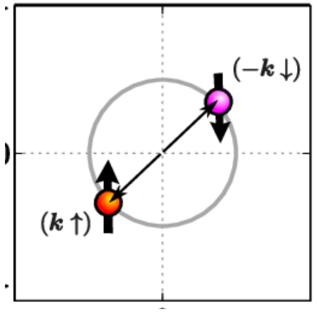
Superconductors:

Discovered, 1911 by Kamerlingh Onnes.

Type-I superconductors:

Bardeen-Cooper-Schrieffer theory:

2 electrons form bound state (mediated by crystal lattice motion):



electron + electron + lattice => bound electron-electron state (Cooper-pair).

electron = fermion

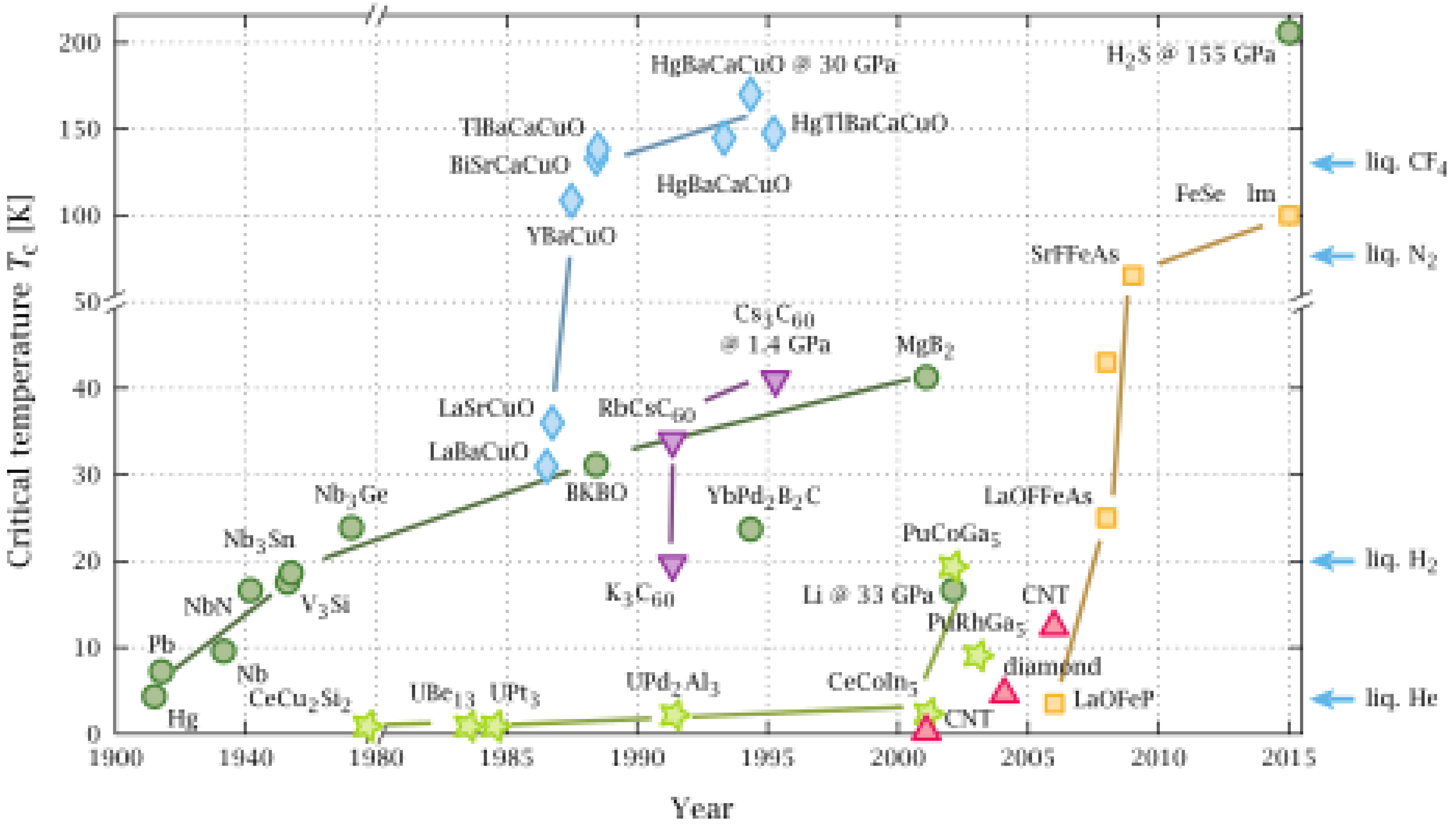
Cooper-pair = boson

Bosons:

Any number of bosons can occupy the same quantum state.

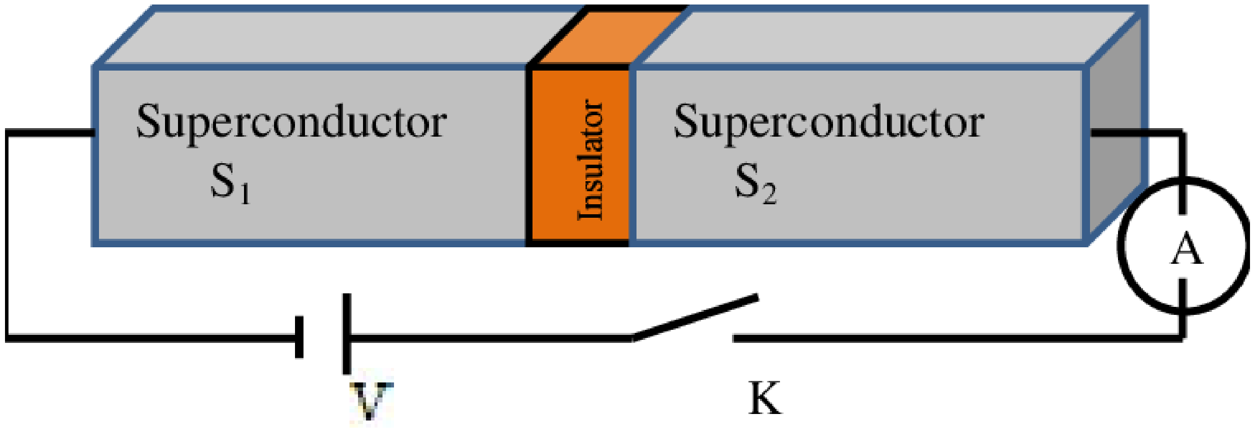
=> Perfect conductor in stability range: zero resistance, NO losses.

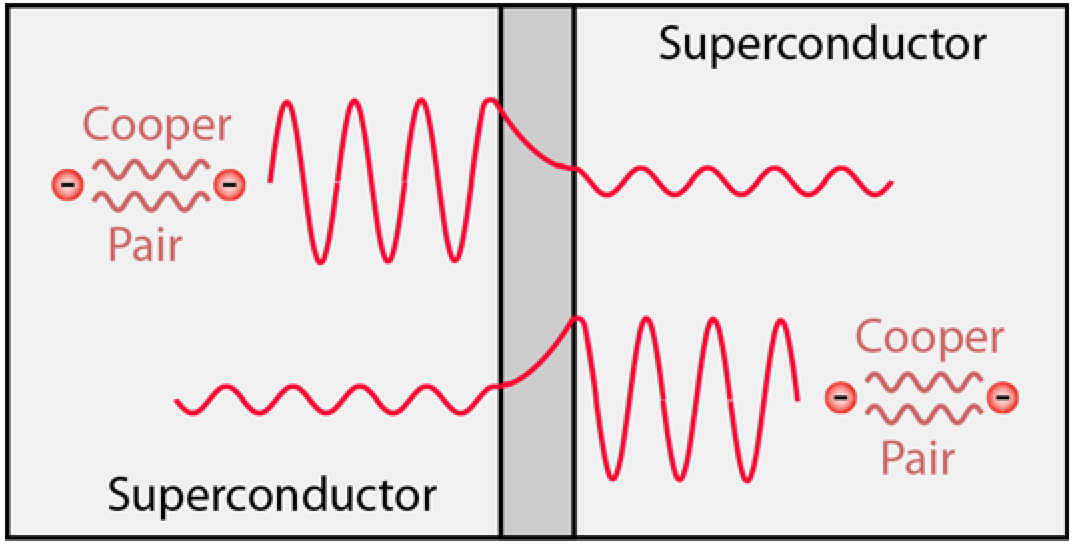
Superconductors – Stability



https://en.wikipedia.org/wiki/High-temperature_superconductivity

Superconductors – Macroscopic Effects
Josephson Junction (JJ)





Tunneling of Cooper-pairs generates supercurrent.

Superconductors – Macroscopic Effects
Josephson Junctions

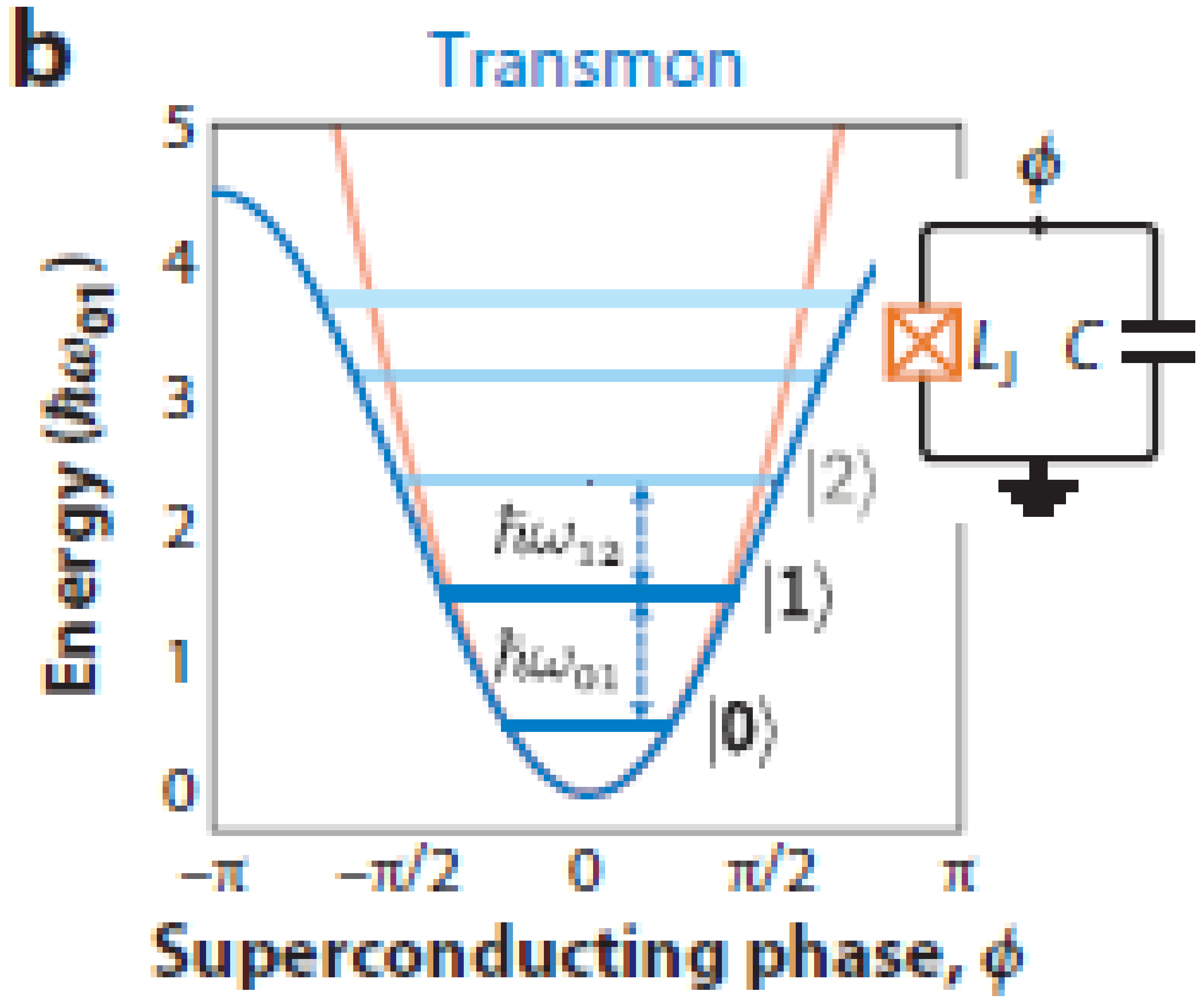
Anharmonic potential:

- NON-equal level spacing.
- Selection of unique qubit possible.

Sensitive to environmental Noise

“Buffer” with capacitor (“C”).

Reduces anharmonicity.



Superconducting 1 Qubit Gates

Supercurrent ➔ can bias with an external applied current.
Spins ➔ can bias with external magnetic field.

$$H = \frac{\bar{Q}(t)^2}{2C_\Sigma} + \frac{\Phi^2}{2L} + \frac{C_\mathrm{d}}{C_\Sigma} V_\mathrm{d}(t)\bar{Q}, \quad (74)$$

Superconducting Qubit/Transmon — Microwave drive

$$H = \underbrace{-\frac{\omega_\mathrm{q}}{2}\sigma_z}_{H_0} + \underbrace{\Omega V_\mathrm{d}(t)\sigma_y}_{H_\mathrm{d}} \quad (78)$$

$$\sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Krantz et al. (2019)

Superconducting 1 Qubit Gates

$$\tilde{H}_{\mathrm{d}} = -\frac{\Omega}{2} V_0 s(t) \begin{pmatrix} 0 & e^{i(\delta\omega t+\phi)} \\ e^{-i(\delta\omega t+\phi)} & 0 \end{pmatrix}. \qquad (90)$$

Superconducting gates are implemented in the time domain.

Consider the interaction part of the Hamiltonian:

$$\begin{pmatrix} 0 & e^{i(\delta\omega t+\phi)} \\ e^{-i(\delta\omega t+\phi)} & 0 \end{pmatrix} = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}$$

Logical operation: must have physical effect => |0> -> |1>, |1> -> |0>

Krantz et al. (2019)

Superconducting 2 Qubit Gates

Connecting TWO transmon qubits leads to a new interaction term:

$$H_{\mathrm{qq}} = g\left(\sigma^{+}\sigma^{-} + \sigma^{-}\sigma^{+}\right) = \frac{g}{2}\left(\sigma_x\sigma_x + \sigma_y\sigma_y\right). \quad (107)$$

Using **time evolution**, we can generate 2 qubit gates.

Krantz et al. (2019)

Superconducting 2 Qubit Gates

$$XY[t] = e^{-i\frac{g}{2}(\sigma_x\sigma_x+\sigma_y\sigma_y)t} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(gt) & -i\sin(gt) & 0 \\ 0 & -i\sin(gt) & \cos(gt) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$XY[\pi/2g] = \text{iSWAP} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

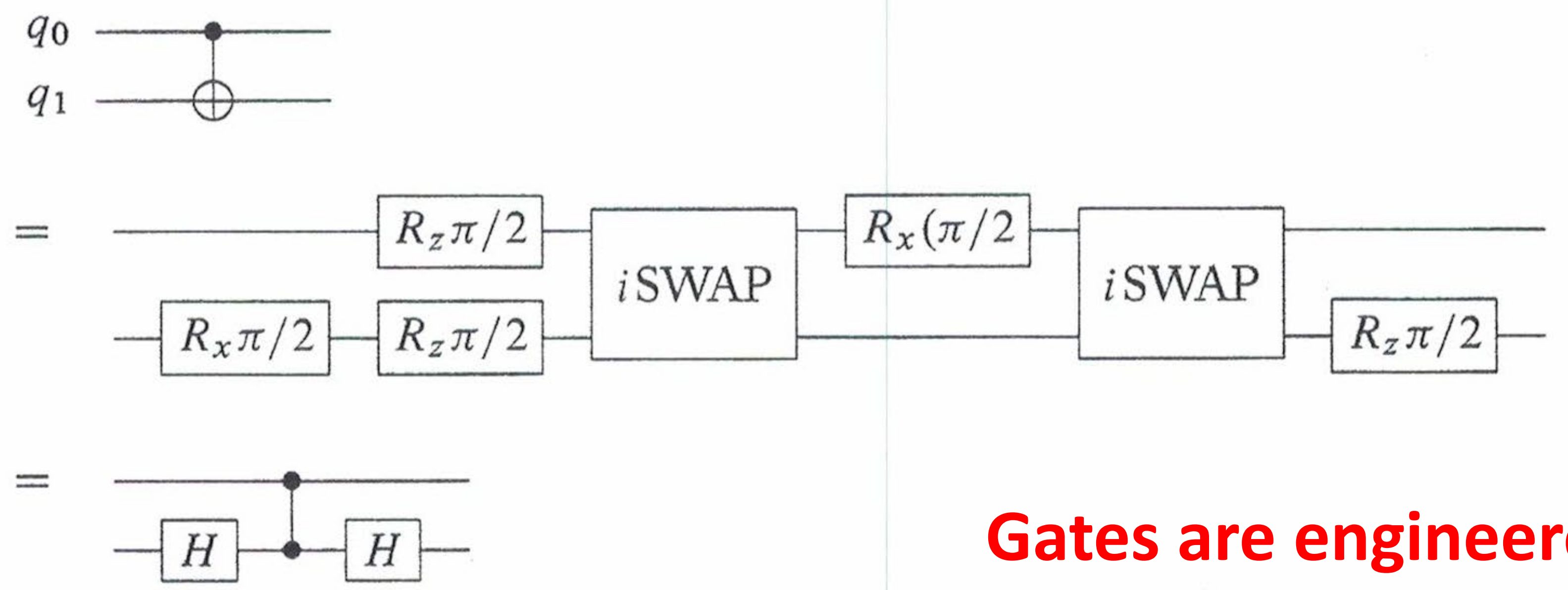
The $\sqrt{i\text{SWAP}}$ gate, which is equivalent to $XY[\frac{\pi}{4g}]$, is sometimes useful as well.

Ding and Chong (2020)

Superconducting 2 Qubit Gate

$$CZ_\theta \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\theta} \end{pmatrix}.$$

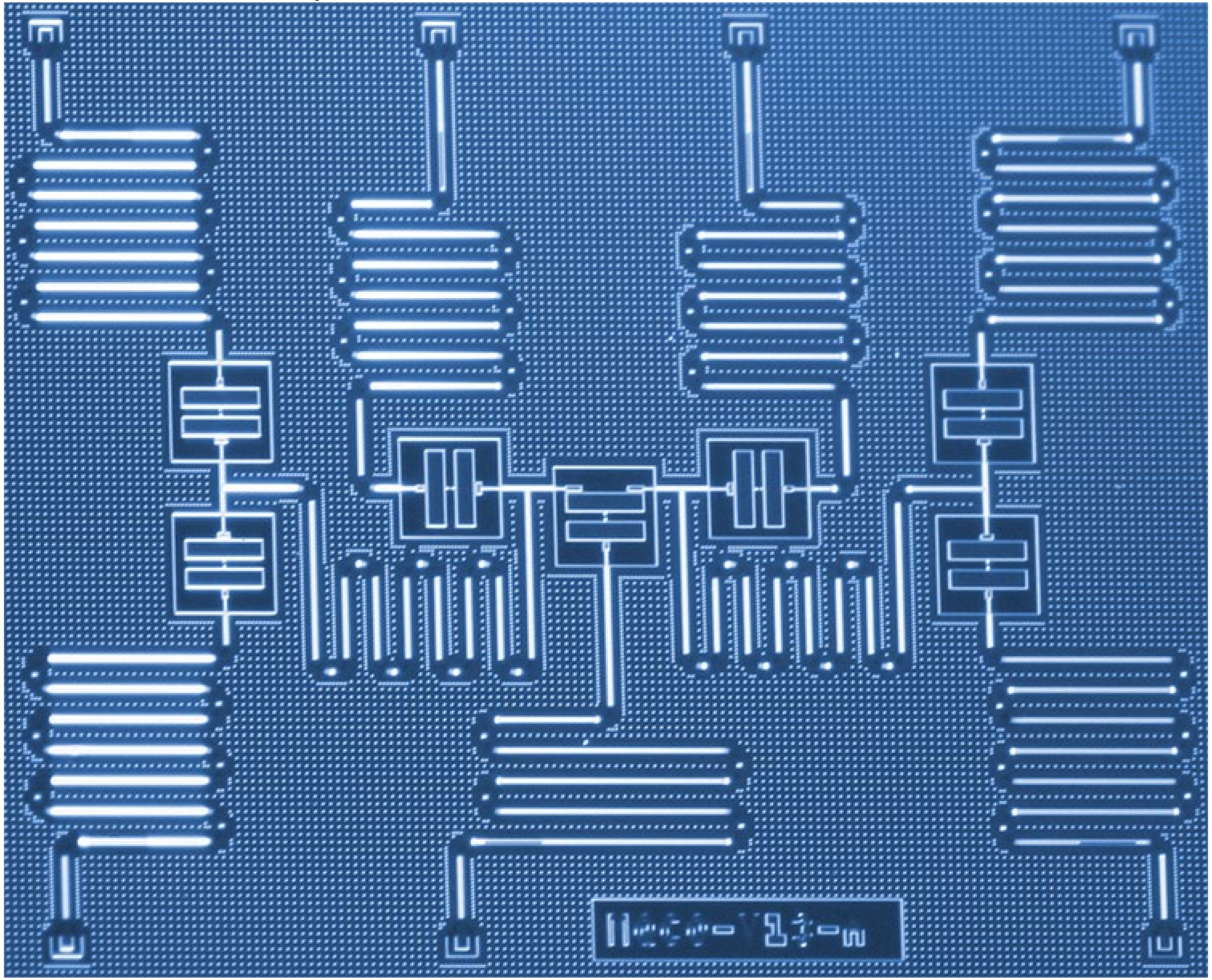
Both the iSWAP and the CZ gates are useful primitives, as they can be used to implement the CNOT gate:



Gates are engineered through the sequence of a several operations in the time domain.

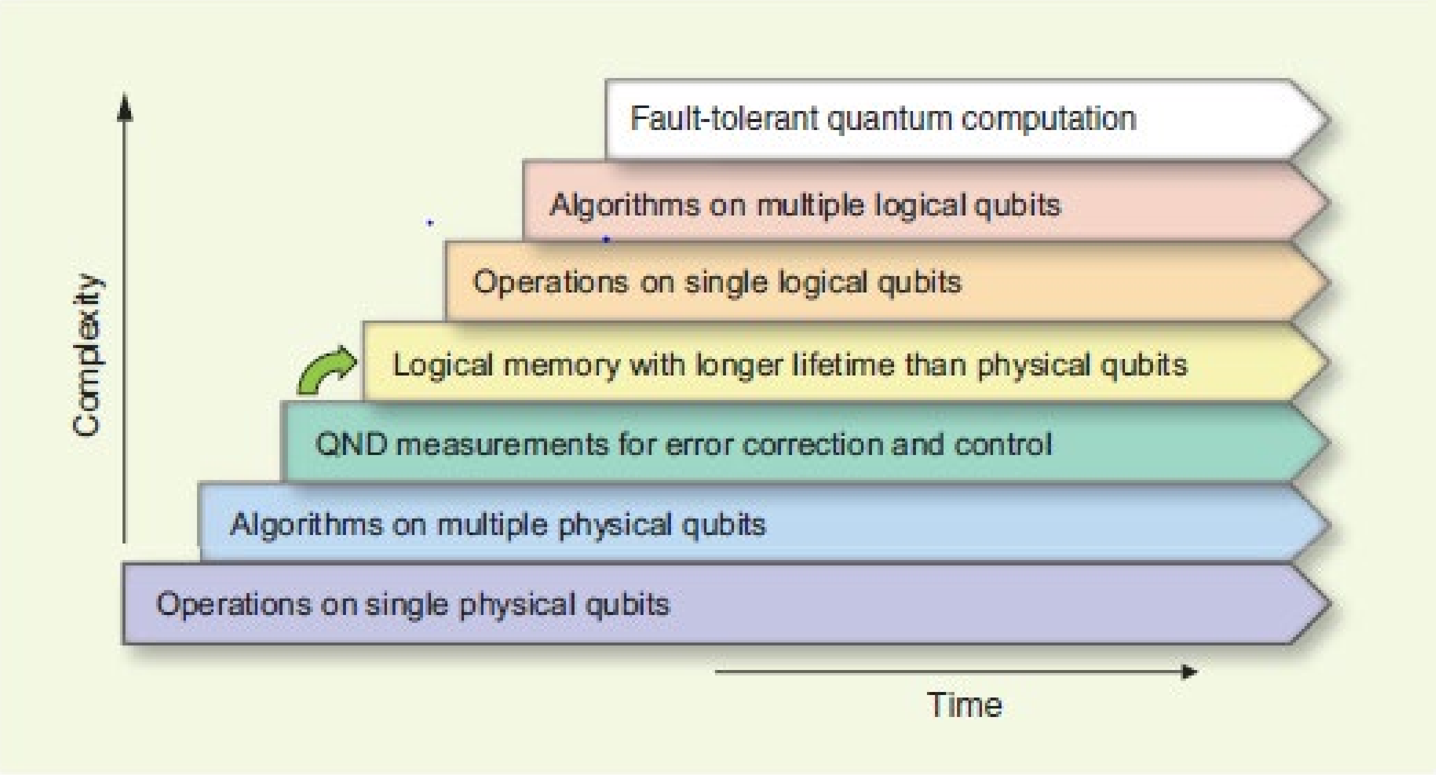
Ding and Chong (2020)

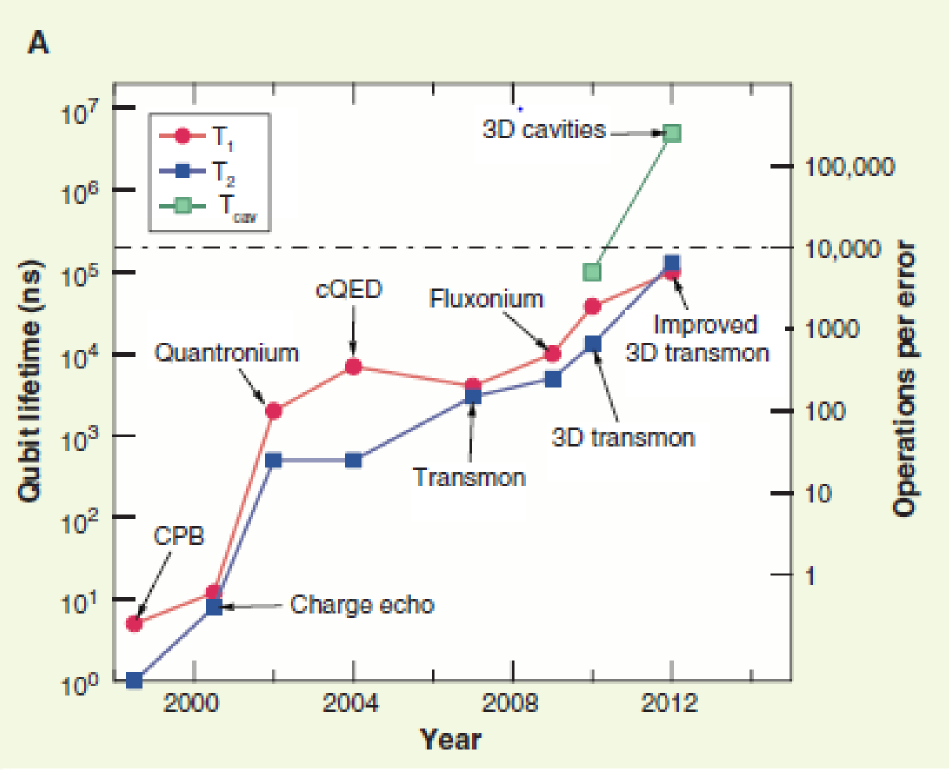
Quantum Processor



https://foresight.org/prototype-quantum-computer-gives-small-molecule-quantum-simulation/

Superconducting Qubits - 2013



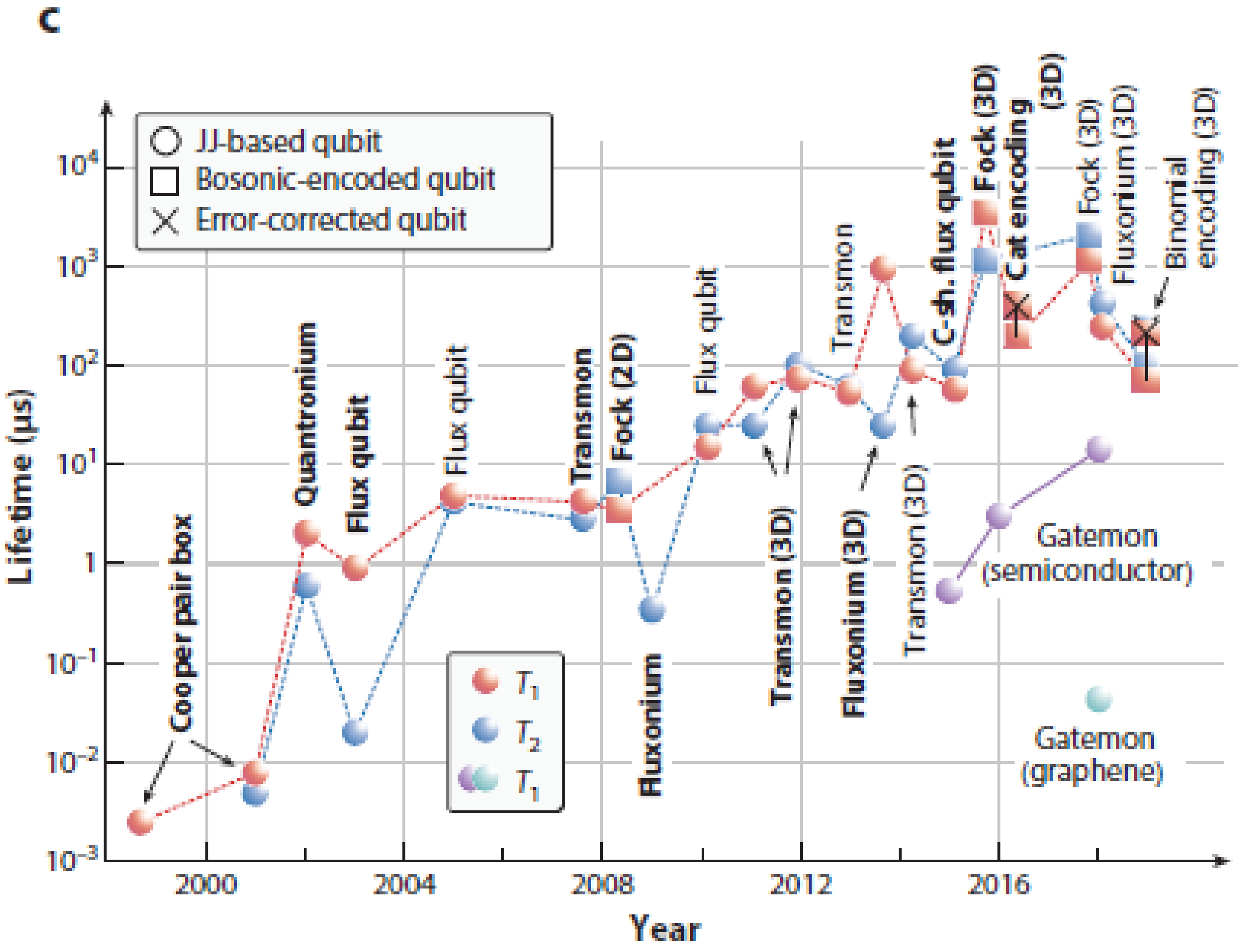


2013: N~2000, crucial to reach stage 4: quantum error correction requires that qubits can be monitored at a rate faster than the occurring error.

Devoret and Schoelkopf (2013)

Superconducting Qubits – 2019

10^3 μs



2013: ~2000

10^5 ns
= 100 μs
~ 10^4 operations

2019:
= 1000 μs
~ 10^5 operations

Hamiltonian Engineering: $$H = \underbrace{-\frac{\omega_{\mathrm{q}}}{2}\sigma_z}_{H_0} + \underbrace{\Omega V_{\mathrm{d}}(t)\sigma_y}_{H_{\mathrm{d}}}$$

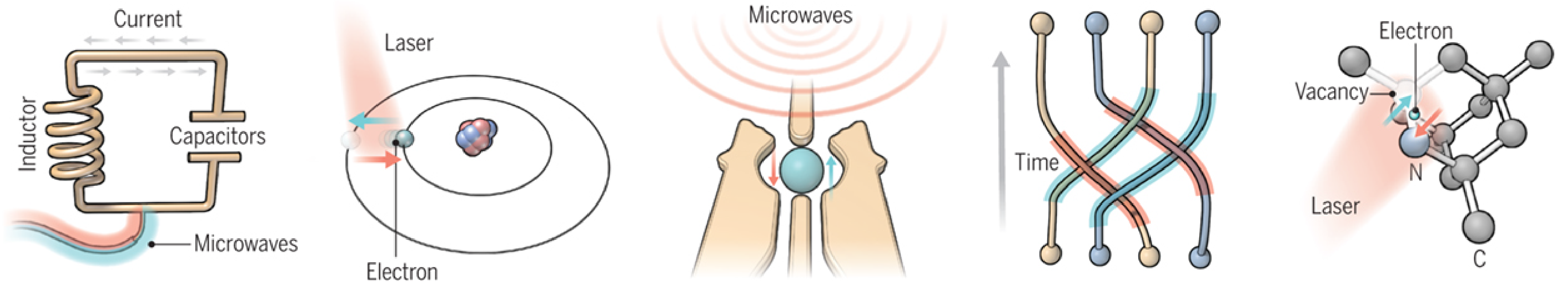
Cross-talk Noise reduction

Krantz et al. (2019)

Kjaergaard et al. (2019)

A bit of the action

In the race to build a quantum computer, companies are pursuing many types of quantum bits, or qubits, each with its own strengths and weaknesses.



	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures. Their braided paths can encode quantum information.	A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.
Longevity (seconds)	0.00005	>1000	0.03	N/A	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%
Number entangled	9	14	2	N/A	6
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

https://science.sciencemag.org/content/354/6316/1090/tab-figures-data

Superconducting Quantum Computers, Theory and Practice

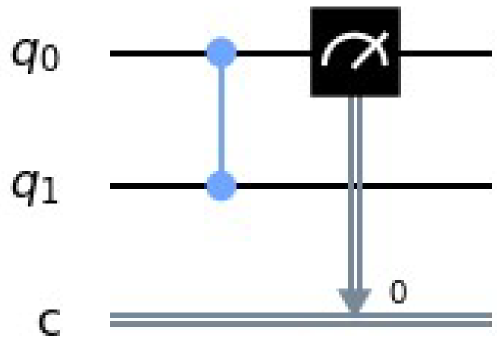
Courtesy: Bryan Garcia (MS, NMSU Physics)

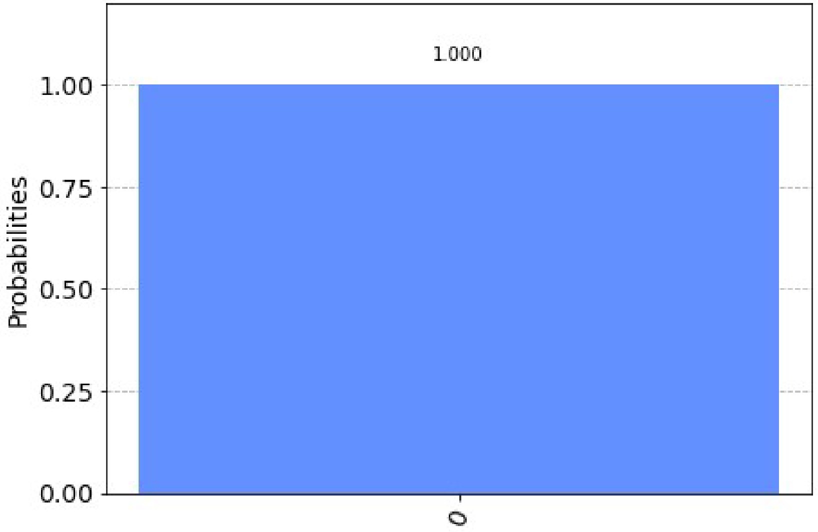
Testing Superconducting Qubits; IBM-Q; CZ-Gate

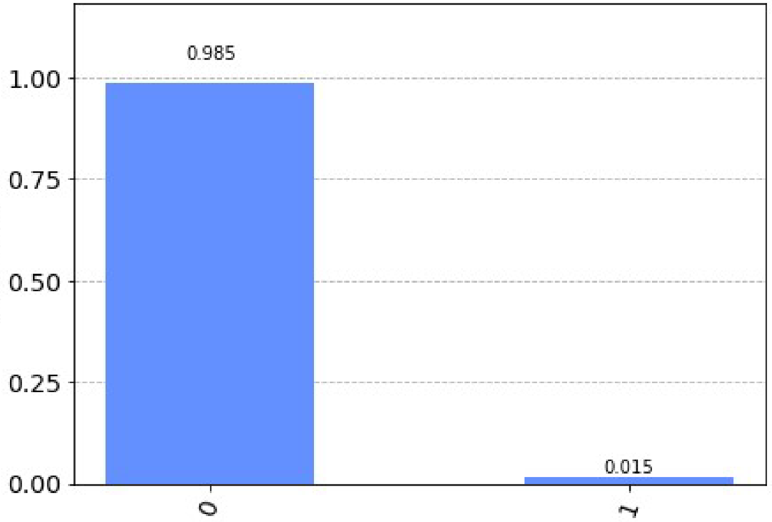
Courtesy: Bryan Garcia (MS, NMSU Physics)

Controlled Z (CZ)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$







Both q_0 and q_1 are in the state |0> by default. In this case we see the state |0> regardless of which qubit we measure, since CZ only induces a phase flip.

Simulator (*Qasm_simulator)*: 100% probability to measure |1>

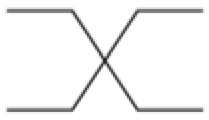
Hardware (*ibmq_16_melbourne*): about a 98% probability to measure |0>

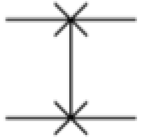
Interested in Quantum Coding? See the IBM-Q developer page https://qiskit.org/

Testing Superconducting Qubits; IBM-Q; SWAP-Gate

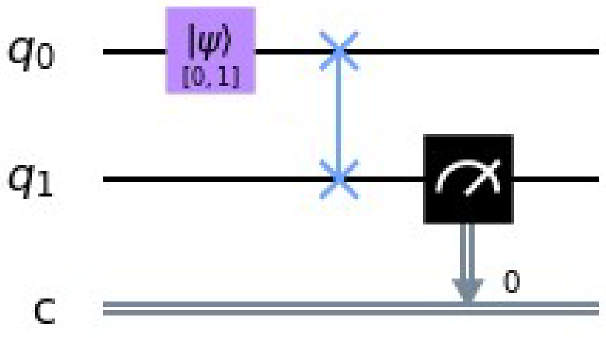
Courtesy: Bryan Garcia (MS, NMSU Physics)

SWAP

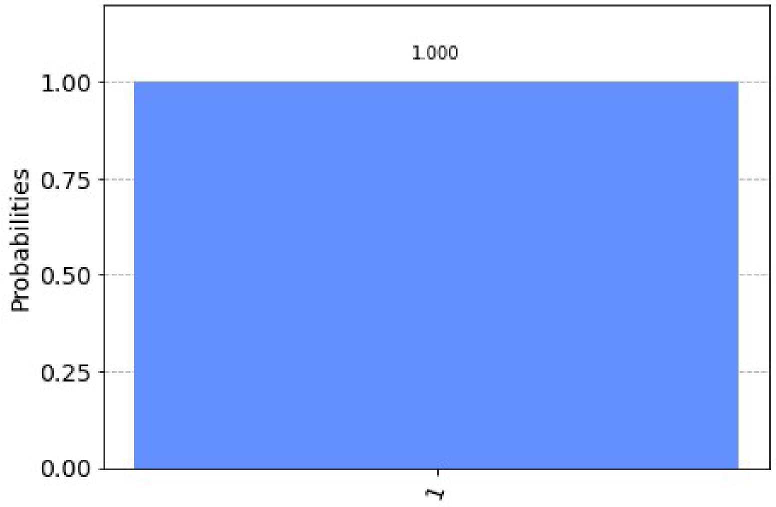




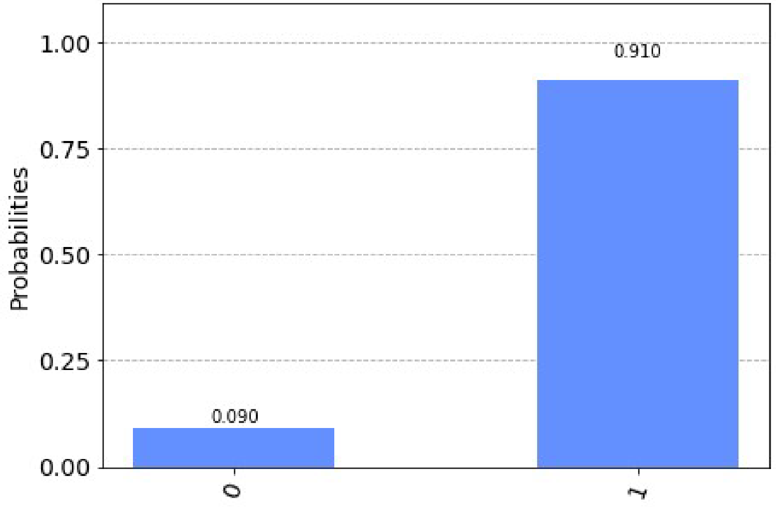
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



q_0 is initialized in the state |1>, we apply a swap gate and measure q_1. We indeed measure the state 1 after swapping



Simulator (*Qasm_simulator)*: 100% probability to measure |1>



Hardware (*ibmq_16_melbourne)*: about a 91% probability to measure |1>

Interested in Quantum Coding? See the IBM-Q developer page https://qiskit.org/

Application
Quantum Communication, Theory and Practice

Courtesy: Bryan Garcia (MS, NMSU Physics)

Quantum Teleportation

Transport information from sender to recipient

Quantum Cloning

Clone information: Quantum version: no-copy theorem => information at sender destroyed.

Quantum Telecloning

Clone information and send to multiple recipients.

Need entangled and equal amplitude states

Examples of Quantum States on a Quantum Processor
Qisket (IBMQ)

(Courtesy: Bryan Garcia, NMSU)

GHZ – States
Dicke - States

GHZ States

- Quantum state entanglement is a crucial element of quantum computing, in fact, it is necessary to implement any efficient quantum algorithm that achieves a computational speedup over a classical computer [1].
- The simplest state that can be entangled is the Greenberger-Horne-Zeilinger state, or the GHZ state.
- A 2-qubit GHZ state (Bells state) has the following state representation:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

- And more generally:

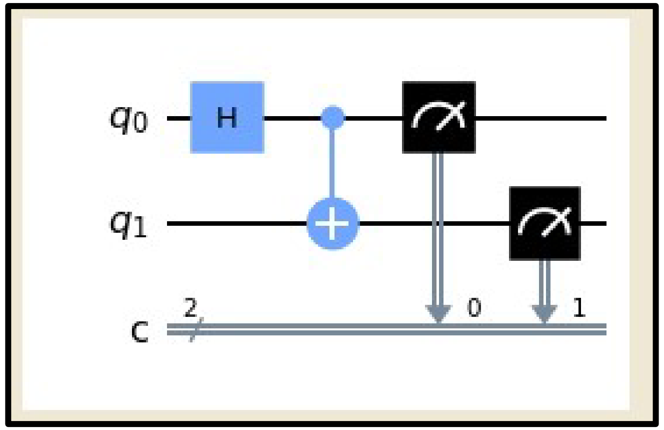
$$|\psi\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j\rangle \otimes \cdots \otimes |j\rangle,$$

with d being the dimension of the system.

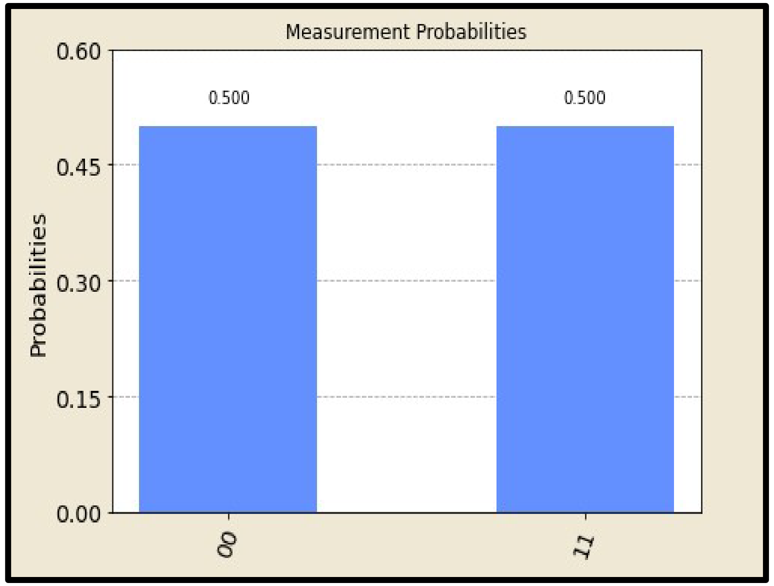
- GHZ states with d=2,3,4 are tested on IBM quantum systems to showcase entanglement on hardware.

2-qubit GHZ State

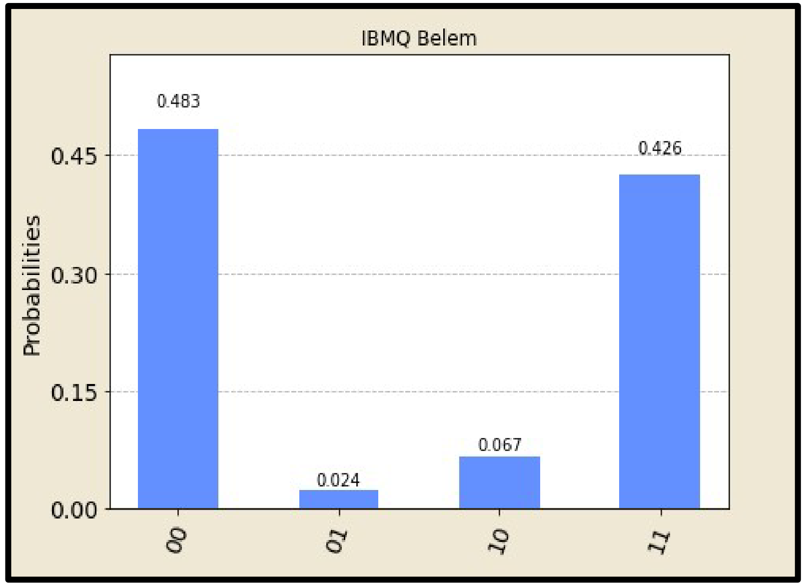
- The 2-qubit GHZ state was executed on the 5-qubit *IBMQ Belem* quantum processor with 15K executions, or shots, of the circuit.
- From left to right, the figures are ordered as circuit, simulator, and hardware:



1) 2-qubit GHZ circuit.



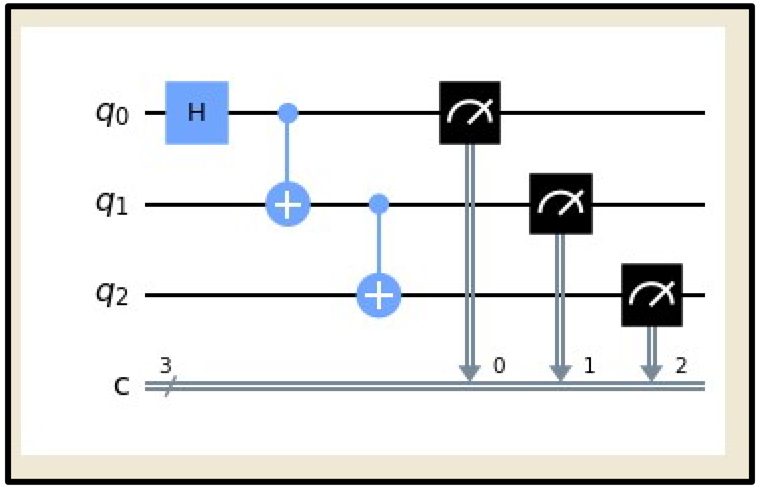
2) Measurement probabilities for 2^2 states in statevector simulator.



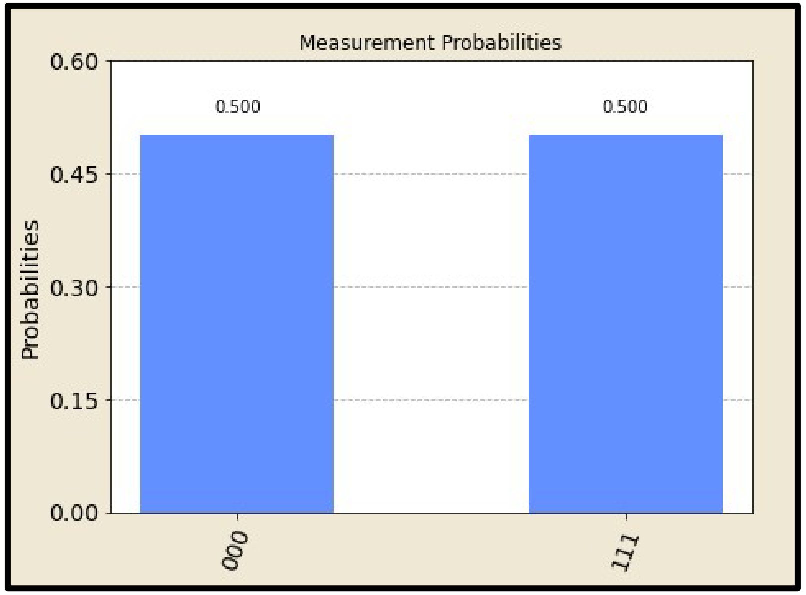
3) Measurement probabilities for 2^2 states on hardware with errors (middle) present.

3-qubit GHZ State

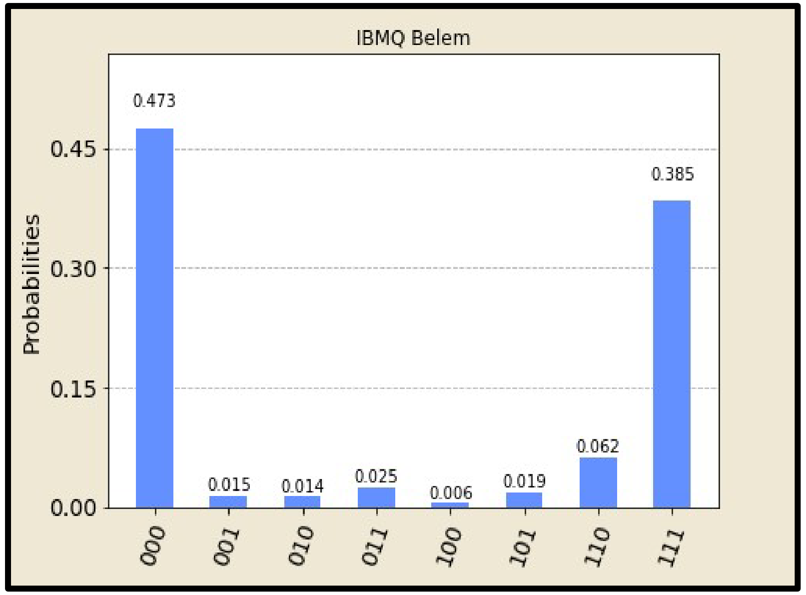
- The 3-qubit GHZ state was executed on the 5-qubit *IBMQ Belem* quantum processor with 15K shots.
- From left to right, the figures are ordered as circuit, simulator, and hardware:



4) 3-qubit GHZ circuit.



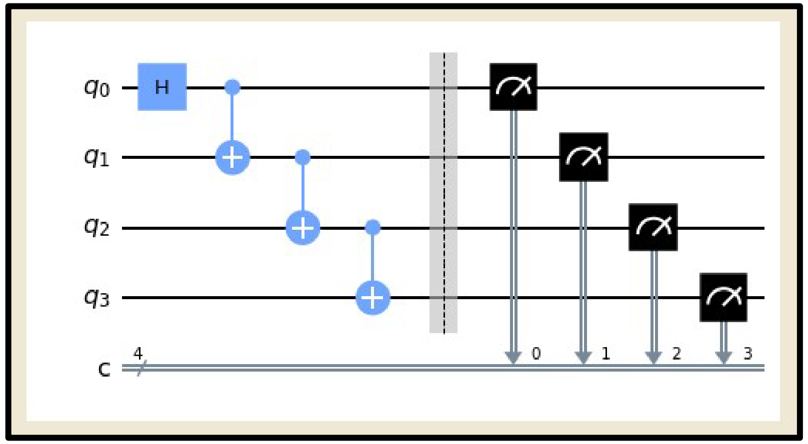
5) Measurement probabilities for 2^3 states in statevector simulator.



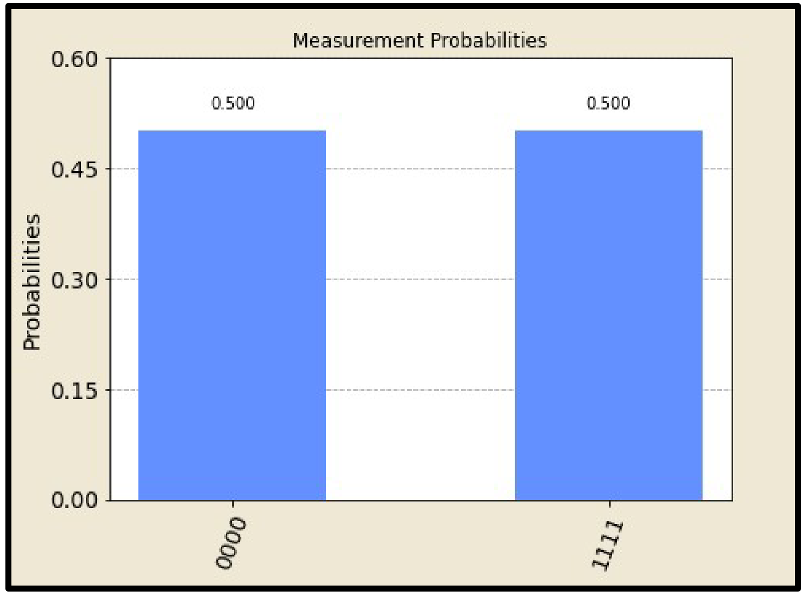
6) Measurement probabilities for 2^3 states on hardware with errors (middle) present.

4-qubit GHZ State

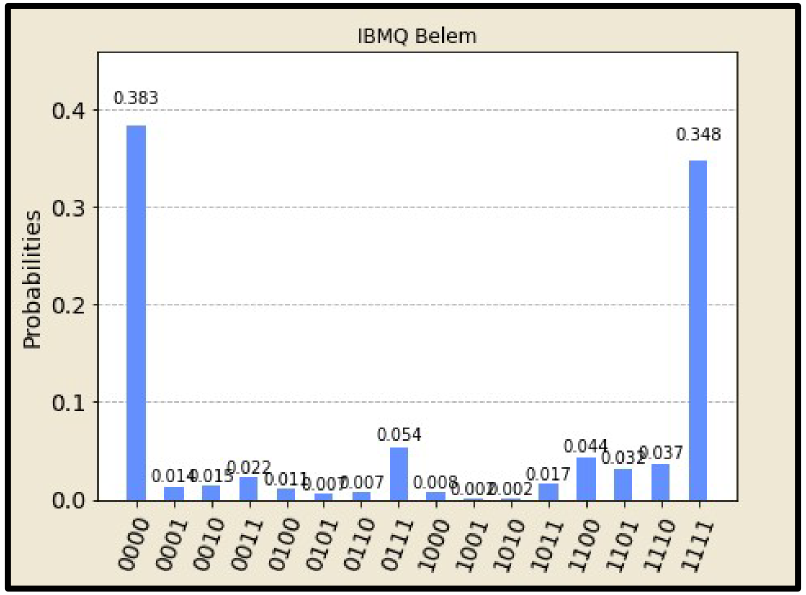
- The 4-qubit GHZ state was executed on the 5-qubit *IBMQ Belem* quantum processor with 15K shots.
- From left to right, the figures are ordered as circuit, simulator, and hardware:



7) 4-qubit GHZ circuit.



8) Measurement probabilities for 2^4 states in statevector simulator.



9) Measurement probabilities for 2^4 states on hardware with errors (middle) present.

Dicke States

- Now we shall look at a more intricate but useful class of states that can be created using entanglement.
- Dicke states are multipartite highly entangled states that are robust to decoherence.
- They are prepared as initial resource states to quantum combinatorial algorithms and quantum communication protocols.
- An n-qubit Dicke state with k excitations is defined as:

$$|D_k^n> = \frac{1}{\sqrt{\binom{n}{k}}} \sum_j P_j\{|0>^{\otimes n-k} \otimes |1>^{\otimes k}\}$$

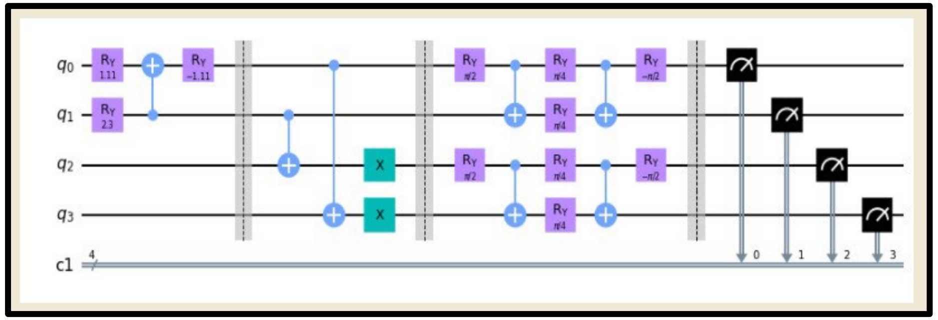
Where $\sum_j P_j\{\cdot\}$ corresponds to the sum over all possible permutations, i.e., for a $|D_2^3>$ state:

$= \frac{1}{\sqrt{3}}(|110> + |101> + |011>)$

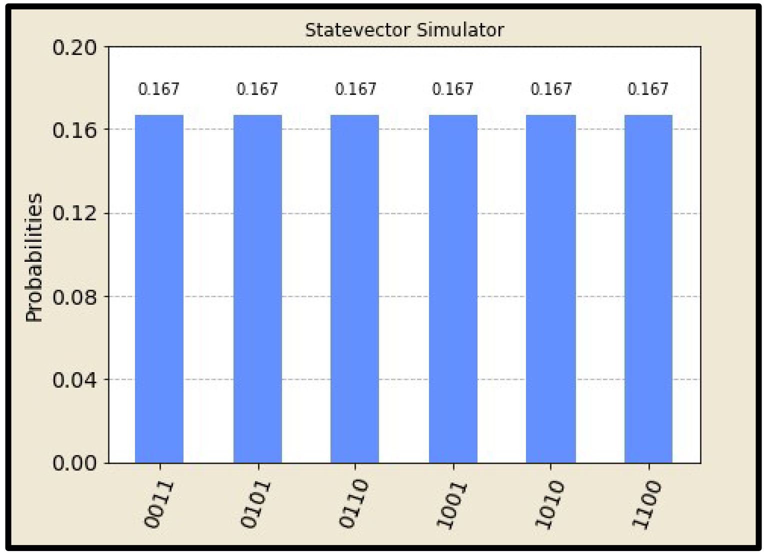
- The Dicke states $|D_2^4\rangle$ & $|D_3^6\rangle$ are tested on IBM quantum systems to show how these multipartite entangled states behave on a real quantum processor.

Dicke State $|D_2^4\rangle$

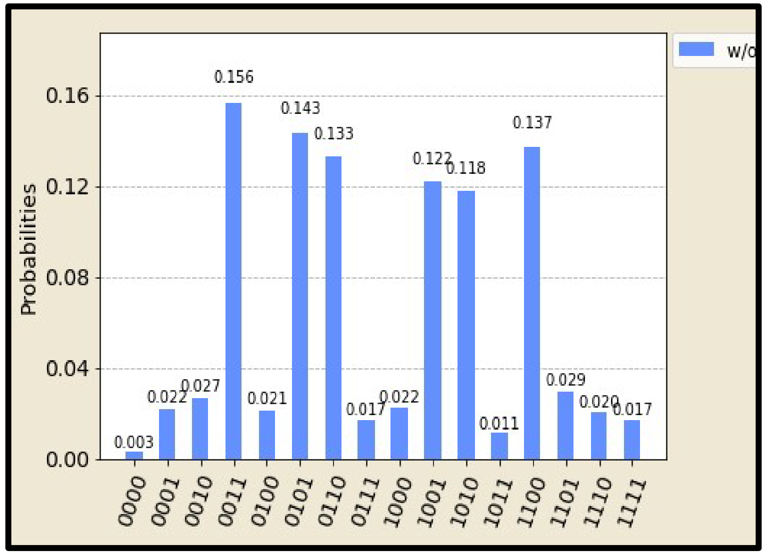
- The $|D_2^4\rangle$ Dicke state was executed on the 27-qubit *IBMQ Montreal* quantum processor with 8K shots.
- From left to right, the figures are ordered as circuit, simulator, and hardware:



10) $|D_2^4\rangle$ Dicke state circuit.



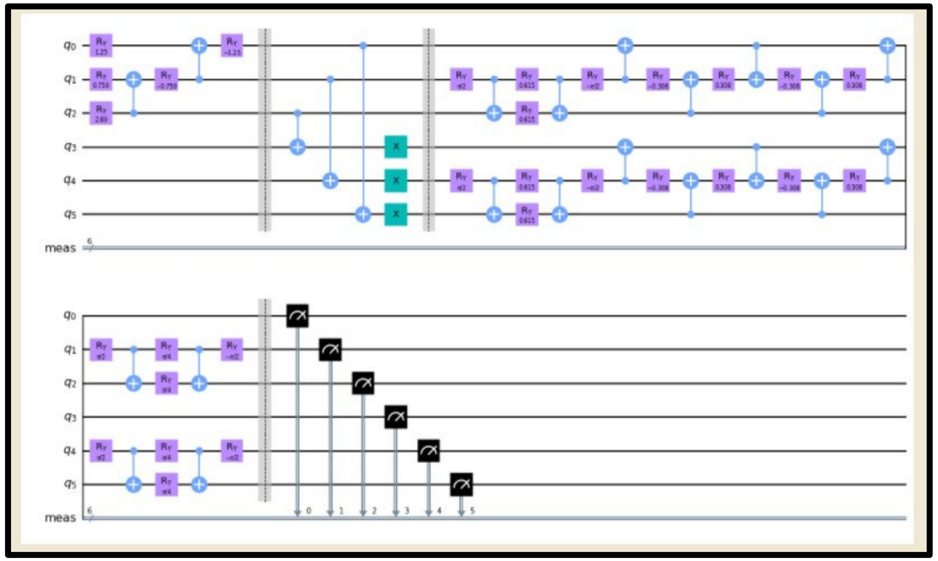
11) Measurement probabilities for 2^4 states in statevector simulator.



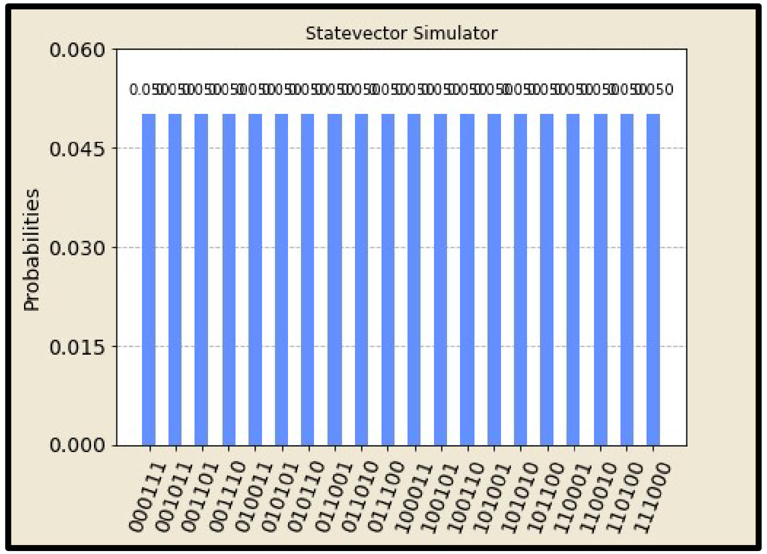
12) Measurement probabilities for 2^4 states on hardware with errors present. The 6 relevant states of the simulator are clearly present.

Dicke State $|D_3^6\rangle$

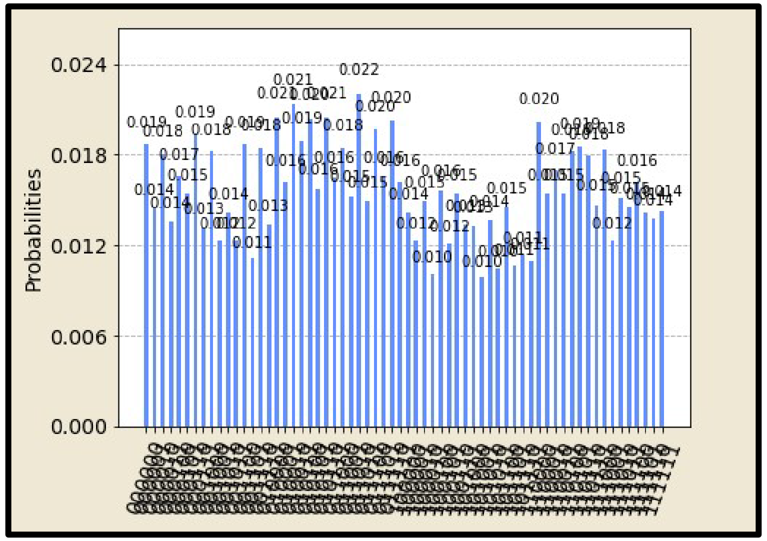
- The $|D_3^6\rangle$ Dicke state was executed on the 27-qubit *IBMQ Montreal* quantum processor with 8K shots.
- From top to bottom and left to right, the figures are ordered as circuit, simulator, and hardware:



13) $|D_3^6\rangle$ Dicke state circuit.



14) Measurement probabilities for 2^6 states in statevector simulator. All 20 states have equal probability of 50%.



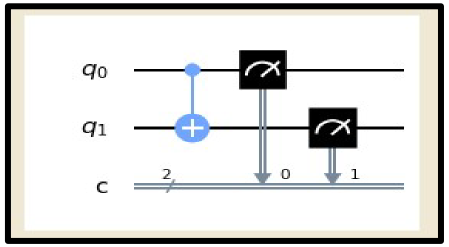
15) Measurement probabilities for 2^6 states on hardware with errors present. The 20 relevant states are indistinguishable from the noise.

Comments

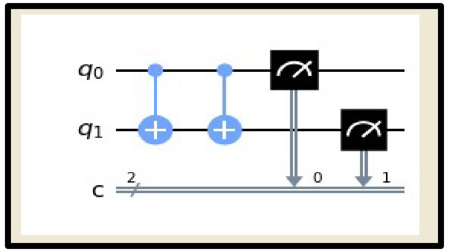
- We have seen the behavior of multipartite entangled states is no different from a GHZ state in that both classes of entangled states have a decline in performance with additional qubits and CNOT gates.
- Now that we have Identified the CNOT transformation as a very noisy gate, the question to ask is in what way can it be used as to not be so harmful to results.
- Is there a distinction to using the CNOT gate in series or in parallel?
- This is tested for stand alone circuits with CNOT’s implemented three times in series and in parallel.

CNOT Gates: In Series, Simulator

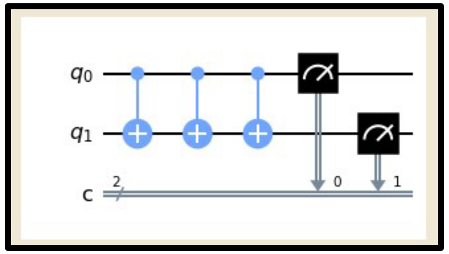
- Here we show the circuits for the CNOT gate implemented 3 times in series and the corresponding expected measurement in the simulator (equal for all three circuits) to the right:



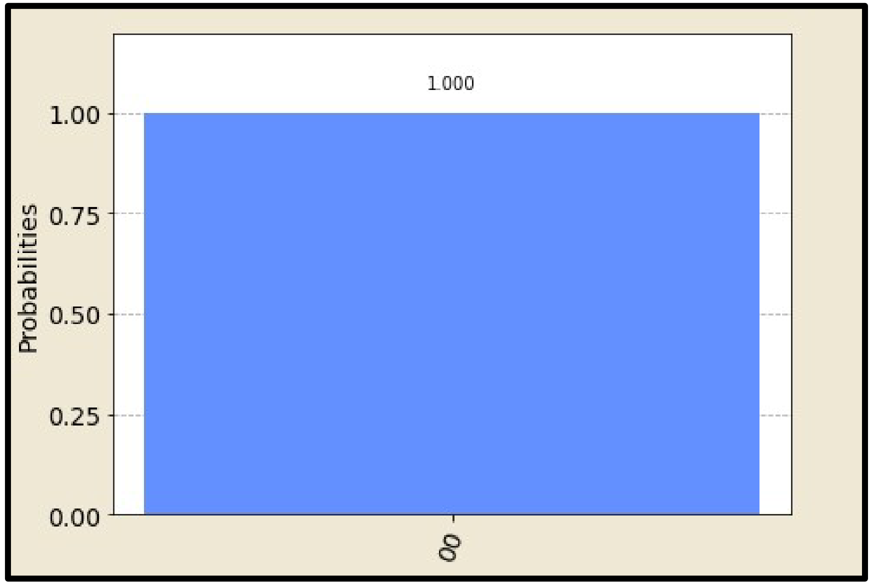
16) CNOT circuit.



17) 2x CNOT circuit.



18) 3x CNOT circuit.

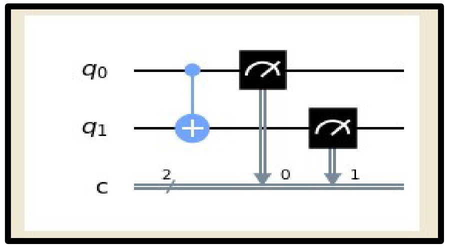


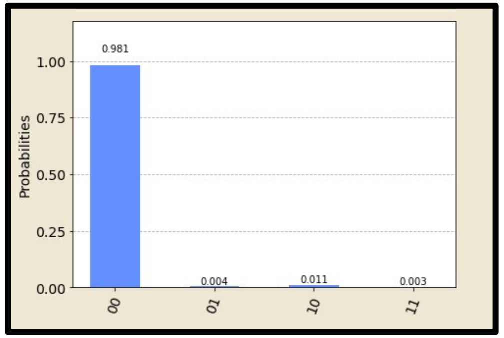
19) Measurement probabilities for all three CNOT circuits in series, simulator.

CNOT Gates: In Series, Hardware

- In the previous slide we saw all three CNOT circuits had the same simulator output.
- Here we test the circuits on hardware and observe if there is a significant drop in performance.

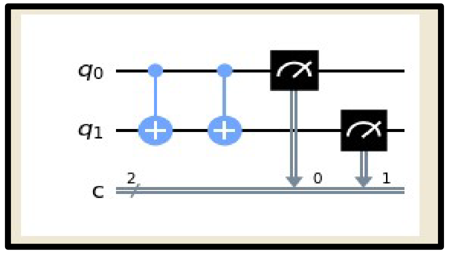
16) CNOT circuit.

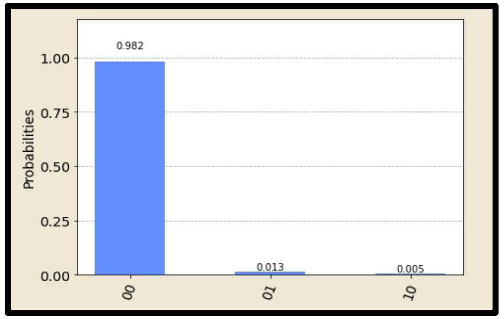




20) Measurement probability success of **98.1%** on hardware.

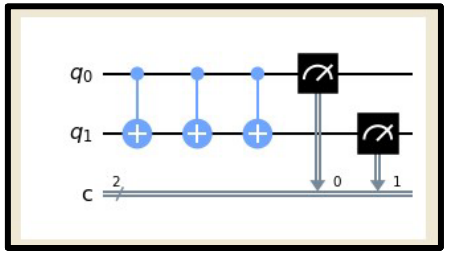
17) 2x CNOT circuit.

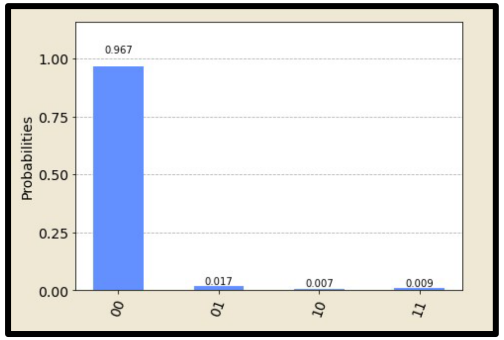




21) Measurement probability success of **98.2%** on hardware.

18) 3x CNOT circuit.

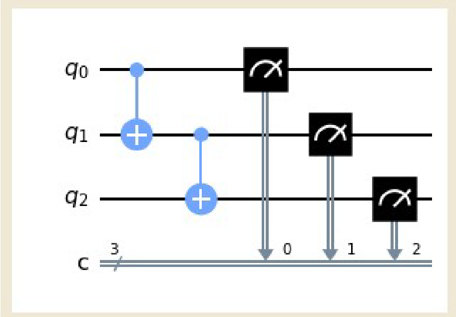




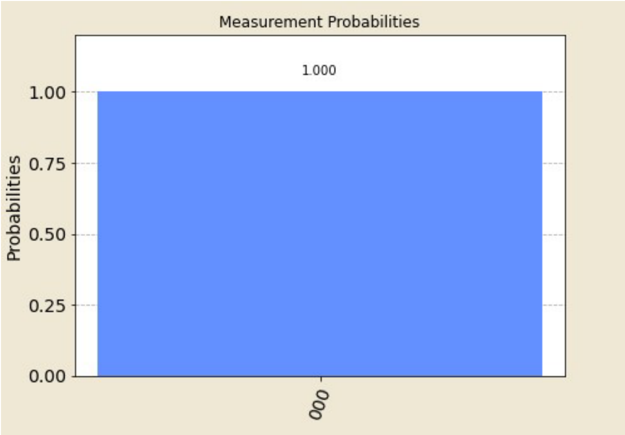
22) Measurement probability success of **96.7%** on hardware.

CNOT Gates: In Parallel, Simulator

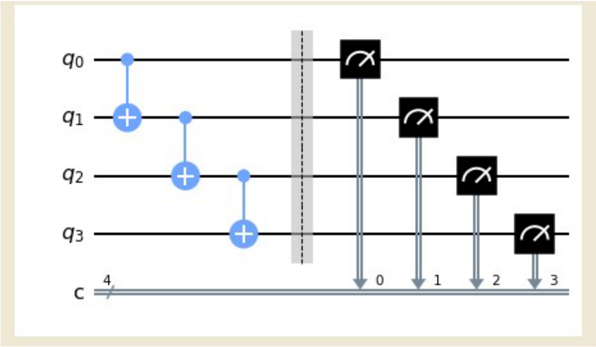
- Here we show the circuits for the CNOT gate implemented 2 times in series and the corresponding expected measurement in the simulator to the right:



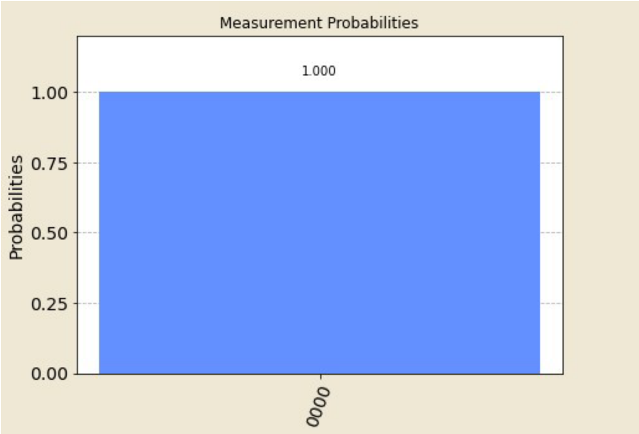
23) 2x CNOT circuit.



25) Measurement probabilities for the 2x CNOT circuit in parallel, simulator.

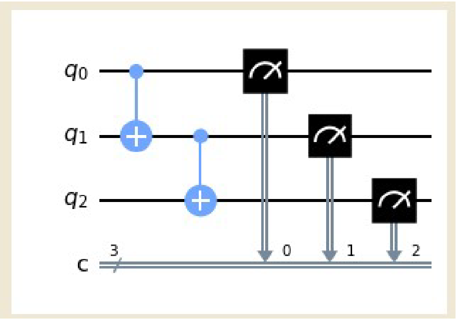


24) 3x CNOT circuit.

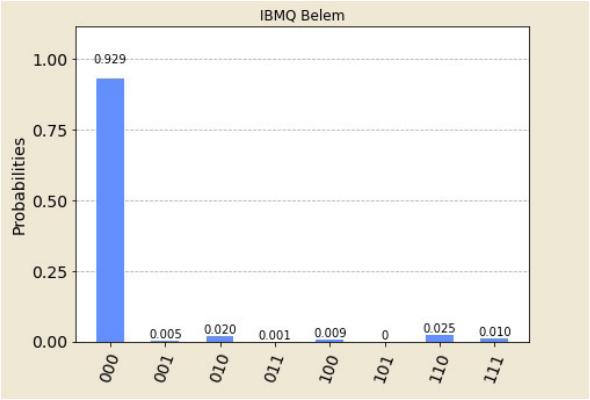


26) Measurement probabilities for the 3x CNOT circuit in parallel, simulator.

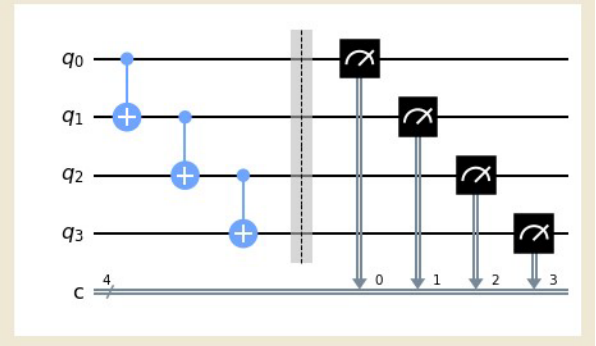
CNOT Gates: In Parallel, Hardware



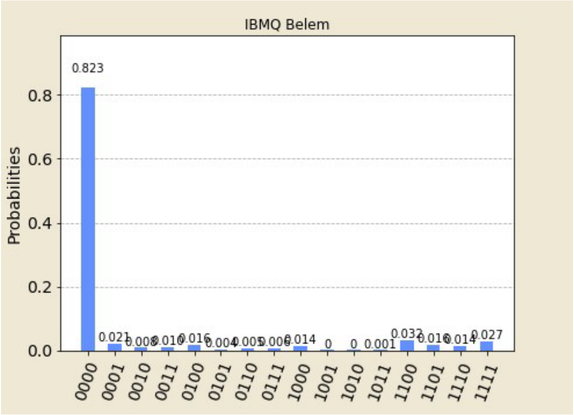
23) 2x CNOT circuit.



27) Measurement probability success of **92.9%** on hardware.



24) 3x CNOT circuit.



28) Measurement probability success of **82.3%** on hardware.

Comments

- As we can see when using the CNOT gate in series there is no significant decrease in performance.
- The measurement probability only drops about 4% when using 3 CNOT gates.
- When we use 3 CNOT gates in parallel we see about an 18% decrease in measurement probabilities. And this is only for a standalone circuit. When we add other gates such as rotations and Hadamard gates, we quickly realize the importance of minimizing the CNOT gates in a circuit.

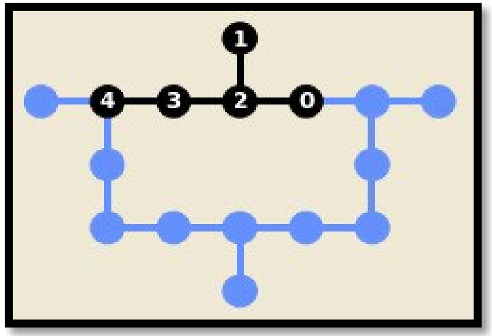
- Not only are gates noisy, but the qubits themselves as well.
- In fact, on hardware, all qubits behave differently due to individual properties like decoherence and relaxation times, readout errors, different frequencies etc. Therefore, it is important to consider which qubits one will be selecting when running a computation.
- In the proceeding section this individual behavior of qubits is illustrated in the fidelities of two telecloned copies, which in theory should be identical.

Quantum Telecloning (QTC)

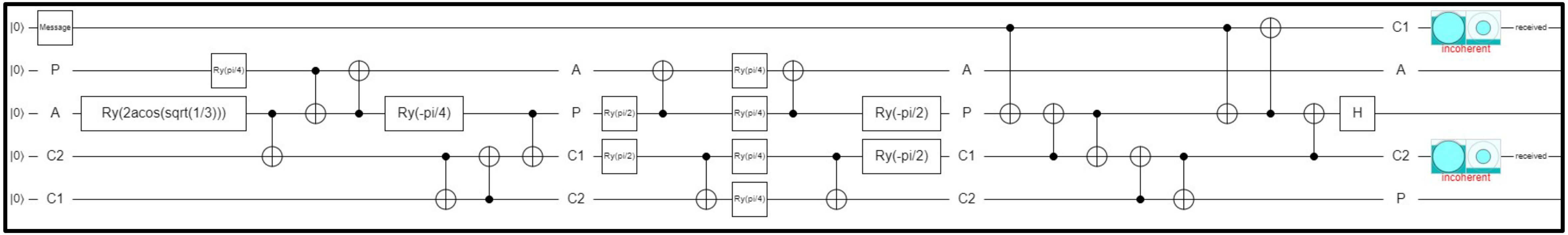
- Quantum telecloning is the generalization of the quantum teleportation protocol to M receivers with N input messages.
- Alice and her party share an initial $2M$ multipartite entangled state, $|\psi >_{TC}$, that serves as the telecloning quantum channel.
- QTC can be implemented in a hybrid quantum system consisting of stationary superconducting transmon qubits (nodes) and optical photonic qubits (links).
- Bi-directional transmon-to-photon conversion [2,3] means that remote superconducting processors can be used for local state preparation and readout nodes in a hybrid quantum network system.
- Here, the deterministic QTC protocol is implemented on a gate-based circuit model, with local operations & classical communication (LOCC) executed via quantum gates.

Current Research: Telecloning (Bryan Garcia)

Virtual Qubits	Physical Qubits
0	10
1	6
2	7
3	4
4	1



Remapping layout on ***Guadalupe***.



Quirk simulator for def. measurement circuit #1 with 18 CNOTs.

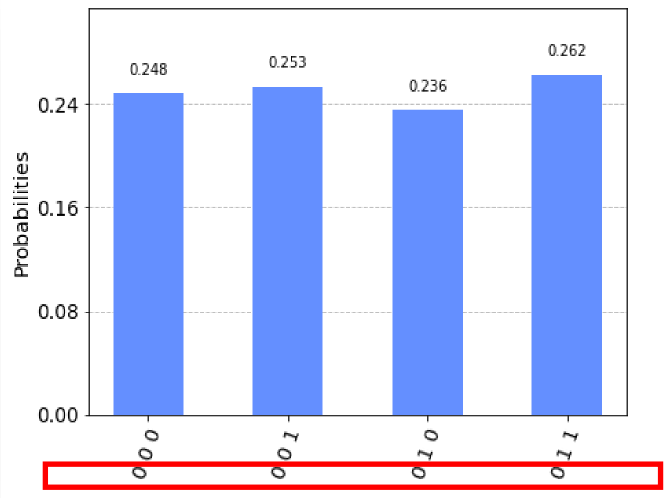
Quantum Communication, Theory and Practice

Courtesy: Bryan Garcia (MS, NMSU Physics)

Simulator (Python based)

```
## Initiazling Circuit
qr = QuantumRegister(3, name="q")
cr1 = ClassicalRegister(1, name="c1")
cr2 = ClassicalRegister(1, name="c2")
qc = QuantumCircuit(qr, cr1, cr2)

## Step 1
# Initializing Alice's q0 to the random state psi
qc.append(init_gate, [0])
qc.barrier()

## STEP 2
# Creating Bell state
create_bell_pair(qc, 1, 2)
qc.barrier()

## STEP 3
# Creating link between q0 and q2 and prepping payload
alice_gates(qc, 0, 1)

## STEP 4
# Alice measures then sends her classical bits to Bob
measure_and_store(qc, 0, 1)
qc.barrier()

## STEP 5
# Bob decodes qubits
bob_gates(qc, 2, cr1, cr2)
qc.barrier()

## STEP 6
# reverse the initialization process
qc.append(inverse_init_gate, [2])
```

Simulator



Bob Measures the state |0> 100% percent of the time

Interested in Quantum Coding? See the IBM-Q developer page https://qiskit.org/

Quantum Communication, Theory and Practice

Courtesy: Bryan Garcia (MS, NMSU Physics)

- **Step a)** Random state to be teleported:

$$|q\rangle = a|0\rangle + b|1\rangle$$

- **Step b)** Alice and Bob each hold a qubit of the entangled Bell state:

$$H\otimes|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$C_{not}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\otimes|0\rangle\right) = C_{not}\left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Unentangled two-qubit state

Entangled two-qubit Bell State

- **Step c)** Applying a CNOT gate followed by a Hadamard gate, the three-qubit entangled system becomes:

$$(H\otimes I\otimes I)(C_{not}\otimes I)(|q\rangle\otimes|\psi\rangle) = (H\otimes I\otimes I)(C_{not}\otimes I)\left(\frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle)\right)$$

$$= (H\otimes I\otimes I)\frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|110\rangle + b|101\rangle)$$

$$= \frac{1}{2}\left(a(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + b(|010\rangle + |001\rangle - |110\rangle - |101\rangle)\right)$$

- **Step d & e)** The state is separated into four states and sent to Bob, which he uses to decode:

$$= \frac{1}{2}\big(\ |00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a|1\rangle + b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)\ \big)$$

$\rightarrow$

Bob's State	Bits Received	Gate Applied
$(a\|0\rangle + b\|1\rangle)$	00	I
$(a\|1\rangle + b\|0\rangle)$	01	X
$(a\|0\rangle - b\|1\rangle)$	10	Z
$(a\|1\rangle - b\|0\rangle)$	11	ZX

$$01: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

[1] Team, T. (2020, December 14). Quantum Teleportation. Retrieved December 2020, from https://qiskit.org/textbook/ch-algorithms/teleportation.html#3.3-Using-the-QASM-Simulator-

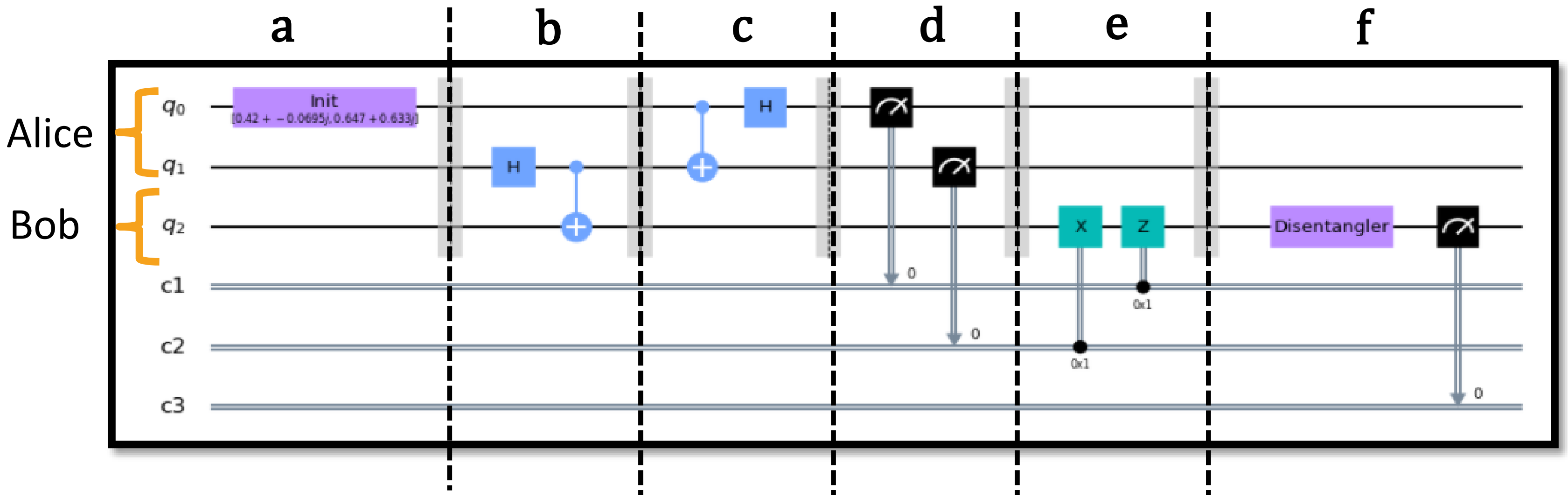
[2] Rieffel, E., & Polak, W. (2014). Ch. 5/Quantum State Transformations. In *Quantum computing: A gentle introduction* (pp. 76-83). Cambridge, MA: The MIT Press.

Quantum Communication, Theory and Practice

Courtesy: Bryan Garcia (MS, NMSU Physics)

Fig.2 **a)** Qubit q_0 is initialized in a random state. **b)** We create a Bell state. **c)** q_0 is entangled with q_1 and q_2. **d)** Alice measures and sends her qubits to Bob. **e)** Bob decodes qubits. **f)** Bob recovers Alice's original state, measures and stores in a classical register.

Qisket: Open-source IBM-Q experience.



[1] Team, T. (2020, December 14). Quantum Teleportation. Retrieved December 2020, from https://qiskit.org/textbook/ch-algorithms/teleportation.html#3.3-Using-the-QASM-Simulator-

Quantum Communication, Theory and Practice

Courtesy: Bryan Garcia (MS, NMSU Physics)

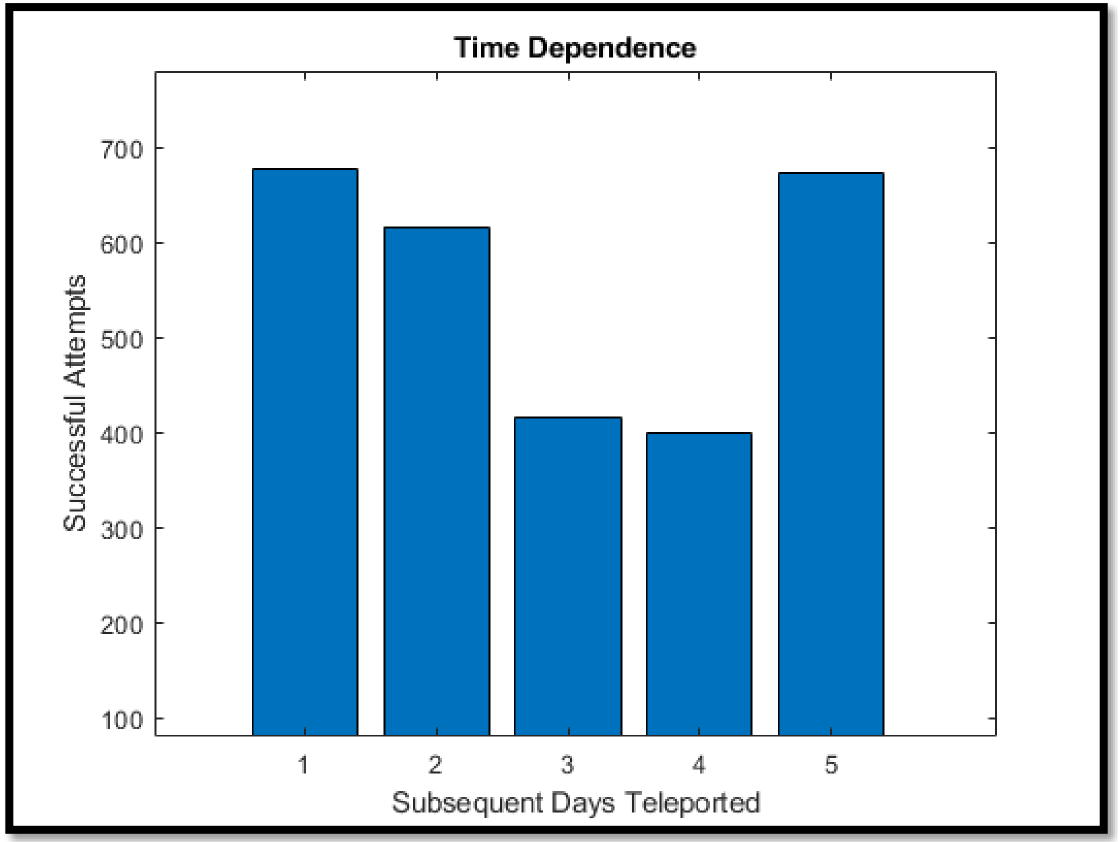


Fig. 7) Circuit was executed at the same time everyday for 5 days from September 25, 2020 - September 29, 2020

Significant daily variations:

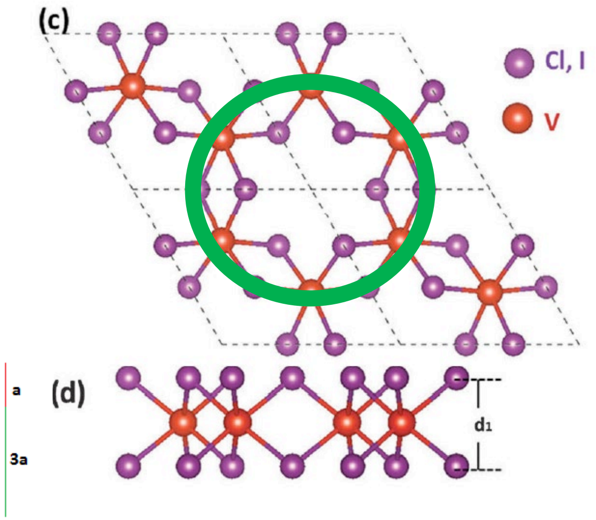
Counting statistics:
Alice sends |0>, how many times does Bob measure |0> as well?

Success rate:
High: ~68%
Low: ~40%

QIS Efforts Kiefer Research Group

Quantum hardware:

- 2D Materials and electron gas
  Discover novel physics.



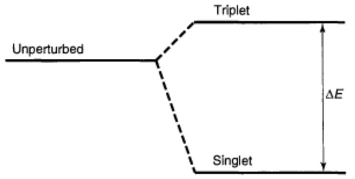
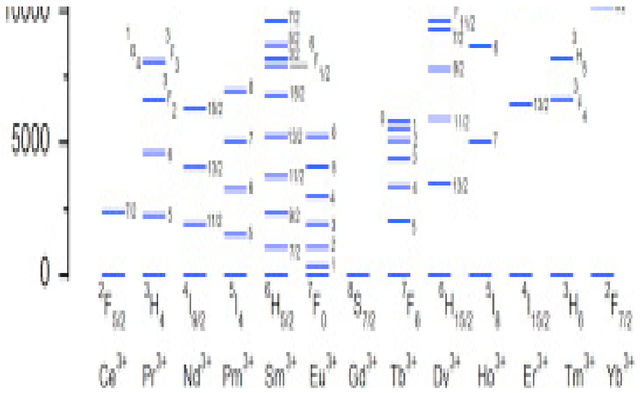
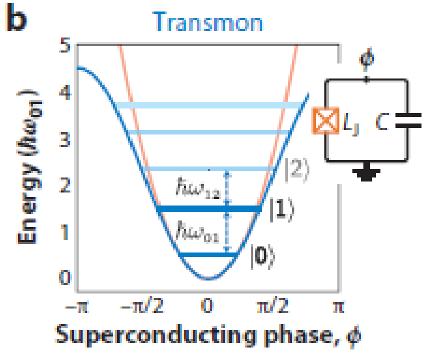


FIGURE 6.13: Hyperfine splitting in the ground state of hydrogen.

- Discovery of novel solid state qubits.



- Molecular qubits for quantum computing.



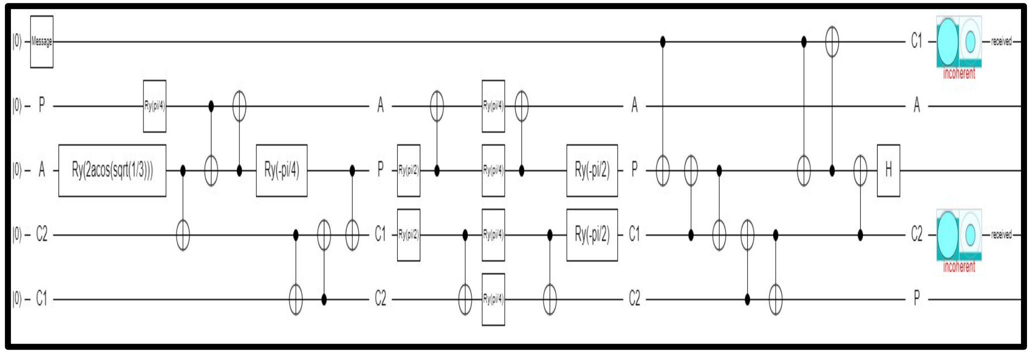
Microwaves: 0.1 – 10 cm^{-1}
(0.01– 1 eV)

- Improving superconducting qubits,
  transmons.

QIS Efforts Kiefer Research Group

Quantum software:

- Multipartite (entangled) states.
- IBM-Q simulator/hardware.
- Next: SNL, ion traps hardware testing.



QISE Workforce Development

How to build a Quantum Skilled Workforce? Discussion:

What would you like to see?
What would help you to consider a QISE career?

Summary

- A brief history of the electron.
  - Discrete energy levels.
  - Spin.
  - Periodic Table.
  - Bosons and fermions.

- Review of Core QISE Concepts:
  - Quantum States; Superposition; Measurement; Entanglement.

- Quantum communication.
- Quantum sensing.
  - Gravitational waves.

Summary

- Quantum computing

  Quantum LC Circuit: Equally Spaced Energy levels.

  Superconductivity: A Quantum State of Matter.

  Josephson Junction: anharmonicity.

    NON-Equally Spaced Energy Levels.

    Time domain computing with superconducting qubits.

  Other approaches.

Summary

- Quantum computing: theory and practice.

    Teleportation.
    Cloning.
    Telecloning.

    Multipartite states: GHZ, Dicke-states.
    Telecloning circuits.

    CNOT: entanglement => needed.
    CNOT: reduce as much as possible.

Quantum Information Science and Engineering (QISE)

This is an exciting time, with many new opportunities for quantum enabled technologies.

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